Monetary Tightening and Inflation in a Bayesian Proportional Hazard Model

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Executive Summary

- This article assesses whether the Federal Reserve's monetary tightening would hasten the cessation of inflationary spells in the United States.

- Excessive inflationary periods are identified and fitted to a proportional hazard model with a collection of time varying covariates, including the Fed Fund rate.

- While conventional proportional hazard model addresses primarily duration time, a feature that is retained in our model, we introduce additionally a regime switching mechanism to capture nonlinearities that may be present in calendar time.

- The two regimes are associated with low and high parameter uncertainty, or the variability of response of the hazard function to individual covariates. We find that the low uncertainty regime *ex ante* dominates about 80% of the time, with the other regime basically related to periods when inflationary spells end.

- Tightening via the Fed Fund Rate indeed speeds up the termination of inflationary sessions, but the increase in the risk of "failure" is moderate most of the time. On the contrary, being able to alter market's perception of future inflation turns out to be a more powerful tool in shortening an inflationary spell.

1. Introduction

1.1 Economists may have different views over the causes and consequences of inflation, but price stability remains a ubiquitous paradigm in macroeconomics[1]. While previous studies have worked on issues concerning welfare cost of inflation and credibility of monetary policy, this paper takes a different route to look into the relationship between inflation and monetary policy. Specifically, we investigate whether monetary tightening would lengthen or shorten the duration of an inflationary spell, after controlling for fundamentals and inflation expectation.

2. The Proportional Hazard Model

2.1 The proportional hazard (PH) model is a general analytical framework widely adopted in the survival analysis literature. *It addresses the probability of failure of an event which in many cases varies with the lapse of time*[2]. The PH model has a convenient feature in that the subject-specific part (containing explanatory variables) of the hazard/failure rate can be separated from the pure time dependent part. A concise introduction to the literature can be found in Winkelmann and Boes (2006).

2.2 Two major differences exist between the standard PH modeling approach and the specification adopted in this paper. First, many studies used partial likelihood to estimate the model parameters, rendering the specification of the baseline hazard unnecessary. The Bayesian approach here allows formal estimation of the baseline hazard, the parameters of which can flexibly encompass the cases of positive, negative and zero duration dependence.

2.3 Second, most applications of the PH model focused on the impact of (duration) time variation and subject heterogeneity on the hazard rate. This paper, on the other hand, addresses also parameter constancy from the perspective of calendar time.

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[1] See, for instance, Galí (2008).

[2] The hazard rate is a function of the time spent in a certain state prior to termination. The time here refers to duration instead of calendar time.

2.4 It is well known that a simple PH model can be estimated using logit upon slight refinement of model assumptions. Whether this can be inherited directly depends on the sufficiency in using the simplistic approach. Preliminary visual inspection of the growth cycles[3] of inflation and the Fed Fund rate (FFR) shows a mixed pattern of relationship. The result from trial logit estimation also indicates a positive role of the policy rate in the hazard function, i.e. higher FFR leads to shorter inflationary spell, but this is not statistically significant.

2.5 Another issue regarding model validity is the potential endogeneity of the explanatory variables. The primary concern here is the possible reverse causality of the duration of an inflationary spell to policy rates. This is dealt with, though not perfectly, by the following assumptions. First, there is no contemporary feedback from the duration to the FFR. Given that inflation is widely considered a lagging indicator, this assumption is not too unrealistic. Second, if monetary tightening has anything to do with rule-based policy (e.g. Mehra and Minton, 2007), a certain extent of interest rate smoothing may be implied, thereby mitigating the influence of possible reverse causality.

2.6 The model we consider takes the form:

$$\lambda(t|\, X(t), \beta, \zeta_i) \equiv \lambda_t = \lambda_0(t) exp\{X(t)'\beta_\tau + \xi_i\} \qquad (1)$$

where t is the duration of a spell, $X(t)$ is the set of explanatory variables (including time varying covariates) for the hazard function λ, and β_τ is the vector of parameters to be estimated. $\lambda_0(t)$ is the baseline hazard function which indicates the unconditional shape of duration dependence. The term ξ_i inside the exponential in (1) is similar to the setting in unobserved heterogeneity models and is defined over inflation episode i. In our context, this is a measure of episode-specific uncertainty in the hazard rate manifested as a normal random variable $\xi_i \sim N(0, \sigma_\tau^2)$. β_τ is the coefficient vector which shows the impact on log hazard per unit change in the corresponding variables X.

[3] The extraction of these inflationary spells will be discussed in the next section.

2.7 The coefficient vector β_τ is time-dependent and can be better understood with the introduction of an auxiliary dichotomous variable:

$$\delta_\tau = \begin{cases} 1 & with\, prob\, \alpha \\ 0 & with\, prob\, 1-\alpha \end{cases} \tag{2}$$

$$\beta_\tau = \delta_\tau \beta_0 + (1-\delta_\tau)\beta_1, \tag{3}$$

$$\sigma_\tau^2 = \delta_\tau \sigma_0^2 + (1-\delta_\tau)\sigma_1^2, \quad \sigma_0^2 < \sigma_1^2. \tag{4}$$

The specifications (2)–(4) describe a switching mechanism across calendar time between two regimes labeled 0 and 1. From the identifying restriction in (4) *regime* 1 *may be regarded as the high parameter uncertainty regime*, as compared to the low uncertainty regime 0. These regimes dictate the set of parameters realized for the covariates, as indicated in (3). α is the *ex ante* probability of a low uncertainty regime; and σ_τ^2 is the variance of ξ_i and is a component of the variance of the coefficient vector under regime *s*.

2.8 As we take a Bayesian approach to estimate the model, additional details about the baseline hazard and the prior distributions of the parameters have to be stipulated. The baseline hazard is derived from the Weibull distribution[4], with the parameters freely determined by the simulation. Specifically, $\lambda_0(t) = exp(\theta)\gamma t^{\gamma-1}$ and $exp(\theta), \gamma > 0$. For this baseline hazard, *duration dependence hinges on the size of* γ*: there is positive (negative) dependence on duration if it is bigger (smaller) than 1, and no duration dependence if it equals 1.* $exp(\theta)$ is the scale parameter of the Weibull distribution and will be subsumed into the exponential term of (1) for estimation.

2.9 Van den Berg (2008) shows how the (log) likelihood of a similar system without regime switching can be derived. For a spell that lasts through $t = J$ months, we introduce another binary variable $y_j = 1$ if $j = J$ and the observation is uncensored. For all other observations and cases, $y_j = 0$. Then, the likelihood of a single spell i can be written as:

[4] The hazard rate is the density function divided by one minus the cumulative distribution.

$$\mathcal{L}_i = \left(\frac{\lambda_J}{1-\lambda_J}\right)^{y_j} \prod_{j=1}^{J} (1-\lambda_j) \tag{5}$$

2.10 The overall likelihood is then the product of $\mathcal{L}_i$ over all measured spells. We assume a uniform prior for $\gamma \sim U(0,5)$. The prior for $\sigma_s^{-2} \sim Gamma(a(2-s), b)$; $s = 0,1$. We set $a = 10, b = 2$. Augmenting $X(t)'\beta_\tau$ with an intercept term[5], we replace this with $Z(t)'\tilde{\beta}_\tau$. Now, $\tilde{\beta}_\tau = \delta_\tau \tilde{\beta}_0 + (1-\delta_\tau)\tilde{\beta}_1$, and $\tilde{\beta}_s | \sigma_s^2 \sim Normal(\mu, \sigma_s^2 \Lambda)$; $s = 0,1$. μ is a vector with elements equal one[6]. Λ is 100 times the identity matrix of corresponding dimension. Finally, the transition probability has a prior of beta, $\alpha \sim Beta(1,1)$.

3. The Data

3.1 Most of the monthly raw data are obtained from the CEIC database, and the monthly spot oil prices (WTI) are from the economagic.com website. The full sample starts from Jan 1947 and ends in Aug 2012. In order to generate a balanced sample, the actual data set used for estimation is smaller and starts from Oct 1956. Inflation is measured by the year-on-year changes in the seasonally adjusted CPI-U. The policy rate is the monthly average FFR.

3.2 While not the perfect benchmark, the yield differential between 10-year and 5-year Treasury Notes/Bonds (constant maturity) is used to proxy inflation expectation as there is a relatively long coverage of their historical values. The aggressiveness of banks is measured by the ratio of loans and leases to Treasury securities, both recorded in the credit items of banks' balance sheets. Inflation volatility and persistence are, respectively, the standard deviation and 1st order autocorrelation coefficient of the inflation in the past two years computed using a 24-month moving window. Figure 1-3 compare U.S. inflation with other economic data used.

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[5] This is the θ of the baseline hazard function.

[6] It turns out that the results are robust to the choice of prior values of the parameter vector μ.

Figure 1: US Inflation, Unemployment and Inflation Expectation

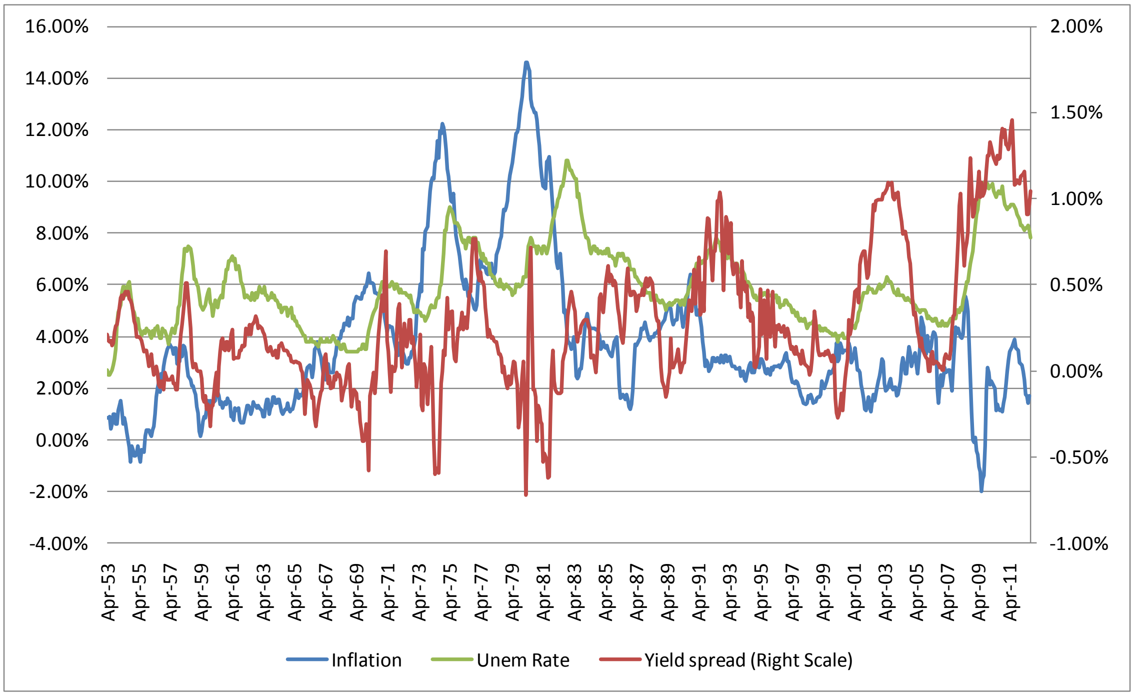


Figure 2: US Inflation and Changes in Oil Price

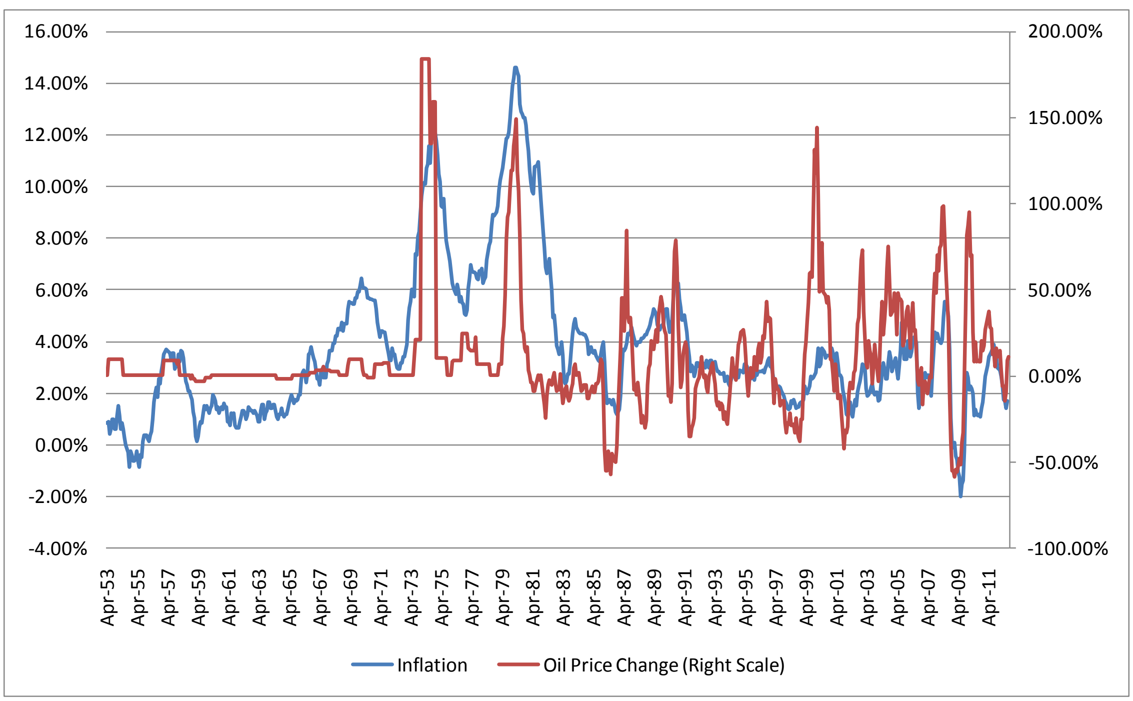


Figure 3: US Inflation and Banking Indicators

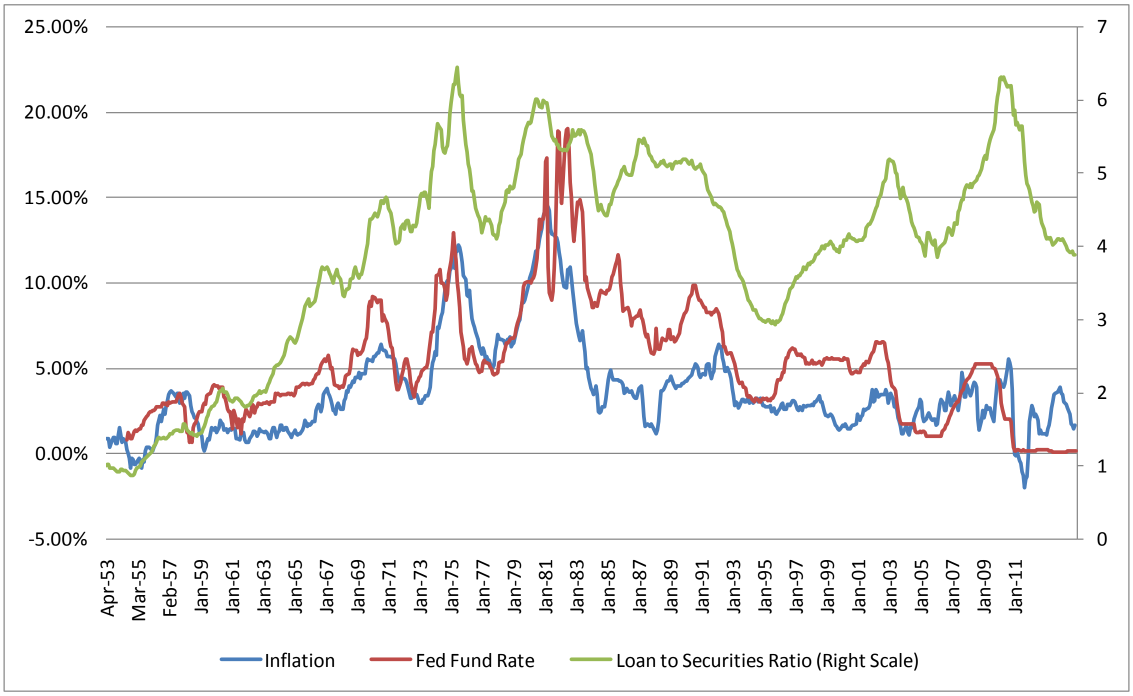
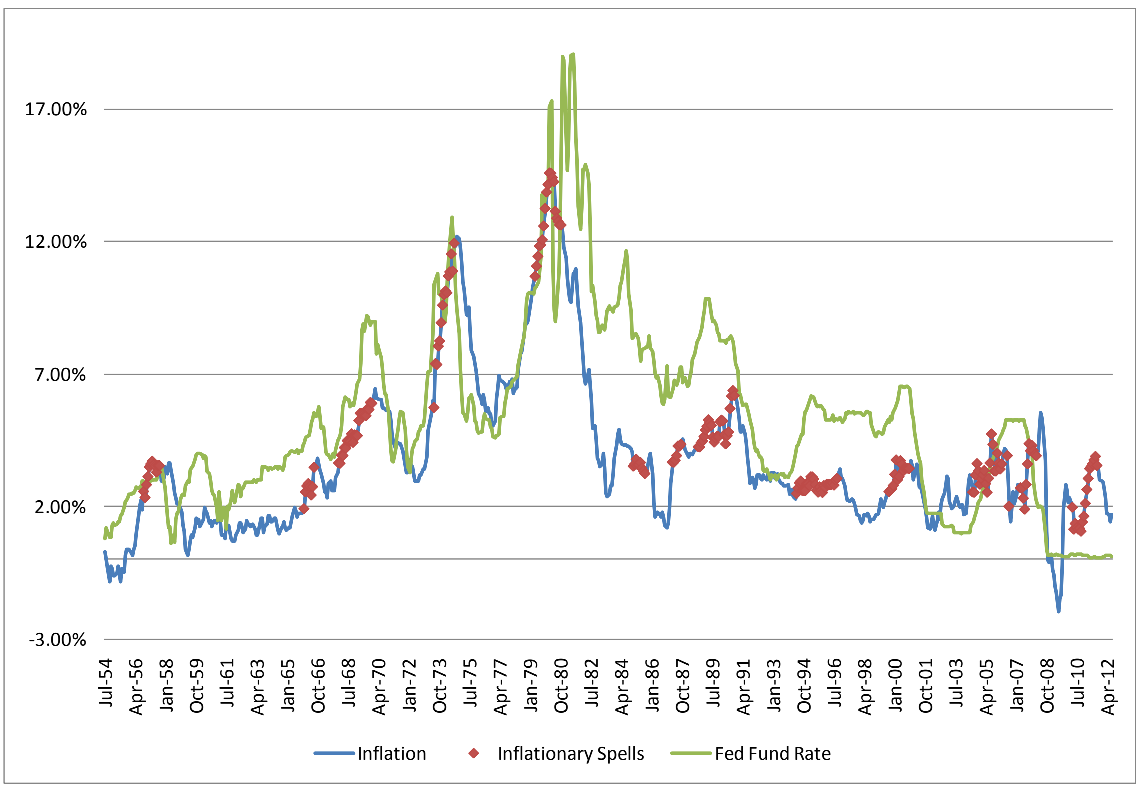


Figure 4: Compiled Inflationary Spells



3.3 Not every point in the time series will be used as our focus is on inflationary spells. So the usable sample contains irregular patches of data representing the various overshooting cycles. Defining an inflationary spell is not going to be uncontroversial given the many signal extraction methods available. Two broad classes of identification schemes commonly used in economics are (i) classical cycles which are episodes of peak-trough movements[7] and (ii) growth cycles which are essentially short run deviations from an underlying trend. The second approach is chosen in this paper because even if inflation bottoms out in a negative territory (hence, away from the trough) it can hardly be regarded as inflationary situation.

3.4 The cycles are extracted using the bandpass filter (Christiano and Fitzgerald, 2003) tuned with signal frequencies of 18 months and 8 years. Excessive fluctuations in the filtered series are then identified using a method similar to Bordo and Jeanne (2002). In brief, if a positive growth figure exceeds 1.25 times the standard deviation of the filtered series within a moving 31-month centered window, it is considered an observation in an inflationary spell. Figure 4 plots the compiled inflationary spells.

4. The Sampling Scheme

4.1 First, note that the hazard rate can be expressed as:

$$\lambda_t = 1 - exp\{-exp\left(X(t)'\beta_\tau + \xi_\tau + \theta + ln(t^\gamma - [t-1]^\gamma)\right)\} \qquad (6)$$

Next, the joint posterior density takes the form:

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$$\pi(\Theta|Data) \equiv f(\beta_\tau, \xi_\tau, \sigma^2, \theta, \gamma, \alpha|Data) \qquad (7)$$

$$= \left(\prod_i \mathcal{L}_i\right)\left(\prod_i f(\xi_i)\right) f\left(\tilde{\beta}_\tau|\sigma^2\right) f(\sigma_0^2) f(\sigma_1^2) f(\gamma) f(\alpha)$$

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[7] See for example Harding and Pagan (2002).

and this will be the basis of target distributions in all Metropolis related simulations. The hyperparameters stated below are selected to achieve a desired level of acceptance ratio for the various Metropolis moves.

4.2 The Shape Parameter γ:

- This is sampled from a Random-walk Metropolis step.
- In iteration m, simulate $\varepsilon \sim Normal(0,0.25)$, and evaluate $\gamma^* = \gamma^{(m-1)} + \varepsilon$.
- With probability

$$\mathcal{A}(\gamma) = min\left\{1, \frac{\pi(\gamma^* | \cdot)}{\pi(\gamma^{(m-1)} | \cdot)}\right\} \tag{8}$$

- Set $\gamma^{(m)} = \gamma^*$, otherwise, $\gamma^{(m)} = \gamma^{(m-1)}$.
- Here, $\pi(\gamma | \cdot)$ is the joint density in (7) with all other parameters held fixed to their latest iterated values.

4.3 The Transition Probability α:

- Let $\Delta = \{\delta_\tau\}$ be the sequence of realized values in an iteration. Let $n =$ dimension of Δ, $n_0 = \sum_\tau \delta_\tau$, and $n_1 = n - n_0$.
- This is drawn from a Gibbs step: $\alpha \sim Beta(n_0 + 1,\ n_1 + 1)$.

4.4 The Uncertainty Parameter ξ_i and σ_τ^2 :

- Draw σ_s^{-2} from the Gamma conditional posterior:

$$Gamma\left(a[s+1] + n_s,\ \left[\frac{1}{b} + \frac{\sum_{\tau:\delta_\tau = 1-s} \xi_\tau^2}{2}\right]^{-1}\right), \quad s = 0,1 \tag{9}$$

- Then, evaluate the sequence Δ such that for each τ,

$$\delta_\tau = \begin{cases} 1 & with\, prob\ \ p_0/(p_0 + p_1) \\ 0 & with\, prob\ \ p_1/(p_0 + p_1) \end{cases} \tag{10}$$

where $p_0 = \alpha\pi_\tau(\Theta|\cdot)$ and $p_1 = (1-\alpha)\pi_\tau(\Theta|\cdot)$, with π_τ being the joint density (7) for the $\tau - th$ observation evaluated with the indicated parameter.

- Draw ξ_i from an independence Metropolis step as follows: Propose for each spell i: $\xi_i^* \sim q(\xi_i^*) = Normal(0{,}0.5)$[8].

- Accept the proposed value with probability

$$\mathcal{A}(\xi_i) = min\left\{1, \frac{\pi(\xi_i^*|\cdot)q\left(\xi_i^{(m-1)}\right)}{\pi\left(\xi_i^{(m-1)}|\cdot\right)q(\xi_i^*)}\right\}, \qquad (11)$$

otherwise, leave $\xi_i^{(m)} = \xi_i^{(m-1)}$.

4.5 Parameters for Time Varying Covariates $\tilde{\beta}_\tau$:

- This is done with a Metropolis Hastings step: Propose for

- $s = 0,1;$ $\beta_s^* \sim q\left(\beta_s^*|\tilde{\beta}_s^{(m-1)}\right) = Normal\left(\tilde{\beta}_s^{(m-1)}, \sigma_s^2\Phi\right)$ with $\Phi = 100 \times (\mathrm{Z}'\mathrm{Z})^{-1}$.

- Accept the proposed value with probability

$$\mathcal{A}\left(\tilde{\beta}_\tau\right) = min\left\{1, \frac{\pi(\beta_s^*|\cdot)q\left(\tilde{\beta}_s^{(m-1)}|\beta_s^*\right)}{\pi\left(\tilde{\beta}_s^{(m-1)}|\cdot\right)q\left(\beta_s^*|\tilde{\beta}_s^{(m-1)}\right)}\right\}, \qquad (12)$$

otherwise, leave $\tilde{\beta}_s^{(m)} = \tilde{\beta}_s^{(m-1)}$.

5. Empirical Findings

5.1 The estimates of the major parameters are summarized in Table 1. The highest probability density intervals (HPDI), the Bayesian equivalence of confidence intervals are also presented. Our results show that the low uncertainty regime (regime 0) generally dominates with a estimated probability of over 80%. In fact, they are associated with most observations except for the end-points of the inflationary spells.

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[8] This can be done by sampling all i terms as a block.

Table 1: Summary of Estimates in the Proportional Hazard Model

	Regime 0		Regime 1	
Parameters/ Coefficients	Coefficients	Highest Prob. Density Interval	Coefficients	Highest Prob. Density Interval
γ	1.3871	(0.4326, 2.3296)	same	same
α	0.8254	(0.58, 0.97)	same	same
β for:				
Fed Fund Rate	0.0445	(-0.5033, 0.5329)	0.1048	(-1.2028, 1.4885)
Loan/Sec Ratio	0.0009	(-0.0104, 0.0118)	0.0006	(-0.0306, 0.0300)
Yield Spread	1.0298	(-2.9640, 5.0212)	0.9878	(-8.4452, 11.0385)
Unemp. Rate	-0.4743	(-1.4212, 0.5089)	-0.8282	(-3.3784, 1.6114)
Oil Price Growth	-0.0115	(-0.0435, 0.0170)	-0.0225	(-0.1108, 0.0544)
Inflation Volatility	0.2762	(-1.8819, 2.0748)	-0.3976	(-5.3728, 4.9067)
σ^2	0.0652	(0.0192, 0.1181)	0.1710	(0.0255, 0.3618)

5.2 Interpretation of the coefficients is in the usual *ceteris paribus* sense. In the low uncertainty regime 0, an increase in the FFR has a very moderate impact on ending an inflation cycle. For a 1% point increase in FFR, there is a $exp(0.0445) - 1 = 4.55\%$ increase in the hazard of ending the inflationary spell. By similar calculations, an increase in unemployment rate by 1% point reduces such risk by about 38%, and an increase in year on year oil price growth by 1% also lowers the risk by 1.1%.

5.3 On the other hand, the high uncertainty regime 1 has a much higher FFR hazard elasticity. It amounts to $exp(0.0445) - 1 = 11.05\%$. A 1% point increase in unemployment rate reduces the hazard by 56%.

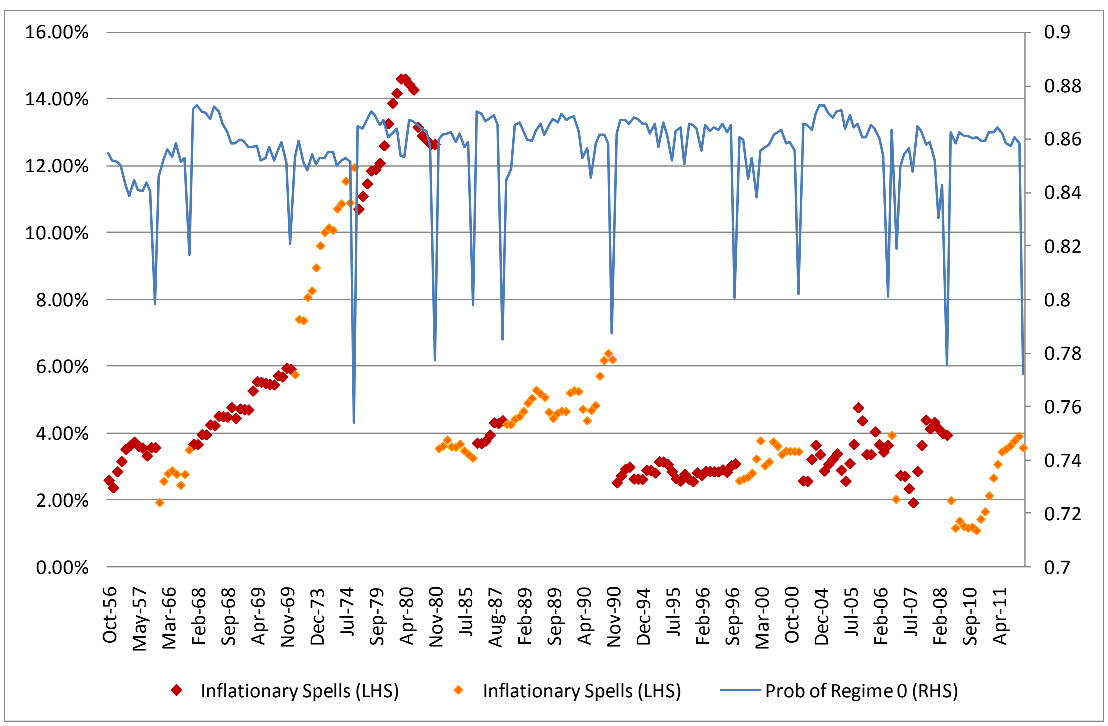
5.4 Yield spread, imposed as a proxy measuring market's perception of future inflation, has a positive effect in hastening the completion of an inflationary cycle. The impact elasticities on the hazard are 1.8 times and 1.68 times respectively in regime 0 and regime 1. So the concern of market seems to be a more important determinant of inflation reversal compared to most other factors included in the study.

5.5 The hazard elasticities of inflation volatilities assume different signs under the two regimes. It is hazard enhancing in the low uncertainty regime, and hazard reducing in the other. This probably reflects the fact that volatility typically increases prior to the end of the inflationary session.

5.6 Recall that the parameter γ governs the degree of duration dependence. Here, the estimated γ is bigger than one, indicating there is positive duration dependence, or the probability that an inflationary spell ends increases with the passage of time.

5.7 Figure 5 plots the inflationary spells and the probability of realizing the low uncertainty regime. The spells are plotted with alternate red and orange colors for easy reference. The time scale is non-continuous and has the time periods not classified as inflationary spells taken out.

Figure 5: Inflationary Spells and Probability of Regimes



5.8 As is evidenced from the diagram, the probability of a regime 0 dips drastically at the end of almost every inflationary spell. In other words, it is the end of each spell that is associated with regime 1.

6. Conclusive Remarks

6.1 A Proportional Hazard Model is fit to US inflation and economic variables to shed light on the determinants of inflation durations.

6.2 The findings are consistent with the evidence of inflation inertia witnessed in certain New Keynesian and Vector Autoregression models.

6.3 In general, there is positive duration dependence, or the probability of an inflationary spell ends increase with its duration. Monetary tightening tends to hasten the end of such sessions, but the impact on the "hazard" is not very strong. Steepening of the yield curve, a proxy for inflation expectation, appears a better predictor for a reversal in inflationary pressures.

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