

The Analytics of Labor Market Matching

Dr. William Chow

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Executive Summary

- This paper reviews an analytical framework that relates unemployment with labor market matching efficiency. It manipulates basically a matching function which takes on information of unemployment and job vacancy and delivers a Beveridge curve in the process. Equilibrium unemployment is determined by movements along, and shifts in, the Beveridge curve (BC) and a so-called Job creation (JC) curve.
- According to the theory, the transition of unemployment goes in counter-clockwise loopings, i.e. there will be a sliding down along the BC (a recession) followed by a rightward shift in the BC as matching efficiency deteriorates in a recession. As the economy recovers, it moves up the new BC before an improvement in matching efficiency finally shifts the BC back to the left again.
- Using data from the General Household Survey and the Survey of Employment and Vacancy, a BC for the period from 2000:Q2 to 2013:Q4 is drawn and the evolution of the unemployment vacancy pair fits the prediction (anti-clockwise loops) of the matching model.
- To quantitatively validate the theory, we estimate a normal time BC and find that the elasticity of matching with respect to market tightness (vacancy per number of unemployed) is about 0.32. Thus, a 1% increase in tightness results in a 0.32% increase in the number of matched jobs.
- As an illustration of the analytics, we apply the method to evaluating the unemployment situation in 2012:Q2, about one year after the implementation of the minimum wage. Based on the given unemployment

and vacancy data and the estimated parameters of the model, we find that the 2012:Q2 matching efficiency is 18% lower than the level in a normal time steady state. This implies an outward shift in the BC which we suspect is attributable in part to the SMW.

- If the cost and benefit of posting vacancies (or hiring) were constant, such a decline in matching efficiency would prompt an equilibrium unemployment rate of 4.95%, compared to the actual 2012:Q2 rate of 3.23%. The discrepancy suggests that instead of unchanged posting cost and benefit, the relative strength of the economic recovery has dampened the cost-benefit ratio and encouraged more employment than what would prevail in equilibrium.

1. Introduction

- 1.1. The discord between Classical and Keynesian economics is well documented. In relation to unemployment, the former asserts that involuntary unemployment should not exist given flexible wage adjustments while the latter argues that market imperfections and wage (and price) rigidities could drive employees with low reservation wages out of the labor market temporarily and involuntarily.
- 1.2. The introduction of NAIRU did bridge the gap somewhat but it is not substantiated by any equilibrium concept (Tobin, 1997). In fact, it is an empirical artifact that explains the inflation-unemployment tradeoff without digging deep into the microeconomic behavior of labor markets.
- 1.3. In the mid to late '80s, an alternative theory emerged ascribing unemployment to frictions in job search, bargaining and matching (Pissarides, 1985; Blanchard and Diamond, 1989; Mortensen and Pissarides, 1994). In a typical search/matching model, an *equilibrium* rate of (frictional) unemployment will be determined primarily by the characteristics of workers and the efficiency of the labor market matching process. "These factors affect the rate at which jobs are simultaneously created and destroyed, the rate of turnover in particular jobs, and how quickly unemployed workers are matched with vacant positions." (Daly *et al.*, 2012)
- 1.4. This paper evaluates the unemployment situation in Hong Kong from the perspective of a search/matching model. Using General Household Survey (GHS) and the Survey of Employment and Vacancies (SEV) data, we trace out the Beveridge curve of normal times and assess whether observed deviations from the equilibrium mix of unemployment and vacancies concur with the prediction of the matching model. The paper is organized as follows: Section 2 introduces search/matching model in brief; Section 3 reviews the Hong Kong data and the basic implications observed; Section 4 discusses how the parameters of the matching function and the Beveridge curve can be estimated; Section 5 explains the empirical findings and Section 6 concludes.

2. Basics of the Matching Model

2.1. A crucial element of the matching model is the matching function which relates the number of job matches (new hires) with the number of unemployed and vacancies. While there are exceptions, the literature is dominated by the Cobb-Douglas type matching function, represented by:

$$H_t = m(U_t, V_t) = AU_t^\alpha V_t^{1-\alpha}, \quad (1)$$

where t is the time subscript; H, U, V denote hires, unemployment and vacancy levels, respectively; A is a parameter indicating the efficiency of the matching process; and α is the elasticity of matching with respect to unemployment. The power of vacancy $1 - \alpha$ is specified in such a way the matching process exhibits constant return to scale¹. We normalize the variables by labor force, thereby converting them into rates which are denoted by lower case letters.

2.2. From this setup, we have the following observations (ignoring the time subscript for quantities defined in the same period):

- The matching function m is increasing in u and v . A fixed proportional increase in them results in an increase in the matching rate by the same proportion.
- **The change in unemployment rate should equal to job destruction rate minus the new hire rate, i.e.,**

$$du_{t+1}/t = s(1 - u_t) - m(u_t, v_t), \quad (2)$$

where s is the separation rate (the rate at which a worker loses his/her employment status and being separated from the pool of employed).

- Market tightness is represented by $V/U = v/u$.

¹ The mechanism of the process is reminiscent of the Cobb-Douglas production used in total factor productivity analysis.

- The job finding rate per unit of time is $h/u = m(u, v)/u$.
 - The job filling rate per unit of time is $h/v = m(u, v)/v$.
- 2.3. In a long run equilibrium, the change in unemployment rate is zero, and from (2) we get the equation of a Beveridge curve (BC),

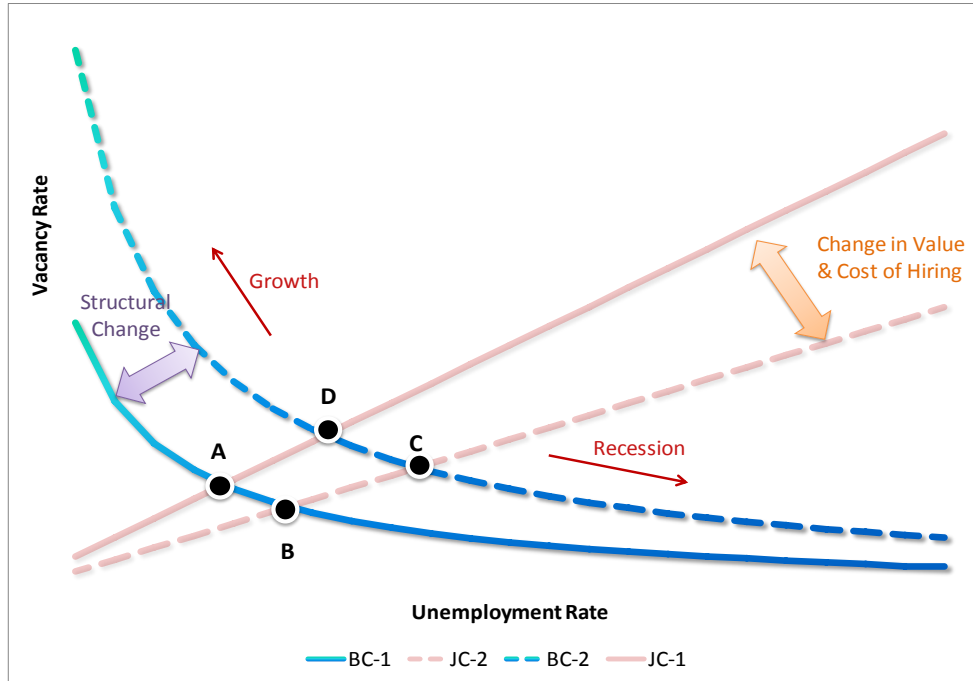
$$u = 1 - \frac{m(u, v)}{s}. \quad (3)$$

Note that **u and v are negatively related along the BC.**

- 2.4. A complete analysis of the labor market search/matching dynamics requires also information on the firms' side, see for instance Mortensen and Pissarides (1994). The cost and benefit analysis of firms' hiring decision can be done comprehensively as in Zanetti (2011). The end result will be the derivation of a so-called "**Job creation curve**" (**JC**) **which is a positive function between u and v** that sums up the information of labor demand. The idea is that the job filling rate in paragraph 2.2 increases with u , so that when the value and cost of hiring remain unchanged firms are willing to open up more jobs as unemployment increases.
- 2.5. **The equilibrium rate of unemployment is determined by the intersection of BC and JC**, as shown in Figure 1. As said, the BC is downward sloping and the JC is upward sloping in the u, v plane. The dynamics of the model can be summarized by the following:
- A movement along the BC implies cyclical shocks – an expansion moves the economy up the BC while a recessive moves it down.
 - A shift in the BC implies structural shocks which affect the degree of mismatch in the labor market, e.g. this could result from a substantial increase in unemployment benefit or from skill mismatch. An outward shift indicates a decline in matching efficiency, and an inward shift indicates the opposite.

- A shift in the JC to the right represents a worsening value-cost mix of hiring, all else equal, while a leftward shift indicates an improved value of hiring vs. cost.

Figure 1. The Beveridge Curve and the Job Creation Curve



- 2.6. What sort of empirical evidence do we have based on the many studies conducted using the matching function approach? It is found that **economies typically experienced anti-clockwise loops around an estimated BC** (e.g. in the sequence A, B, C, D in the diagram) because unemployment adjustments to new postings near the end of recessions are usually sluggish.

3. Overview of Hong Kong's Unemployment

- 3.1. In this section, we review the aggregate unemployment situation of Hong Kong since year 2000. The labor market data are extracted from the official GHS and SEV. Other economic data are collected from the C&SD website. Figure 2 plots the raw data of unemployment rate, the vacancy rate, the growth in real GDP (RGDP) and growth in real wages.

Figure 2. Raw Data of Unemployment and Vacancy

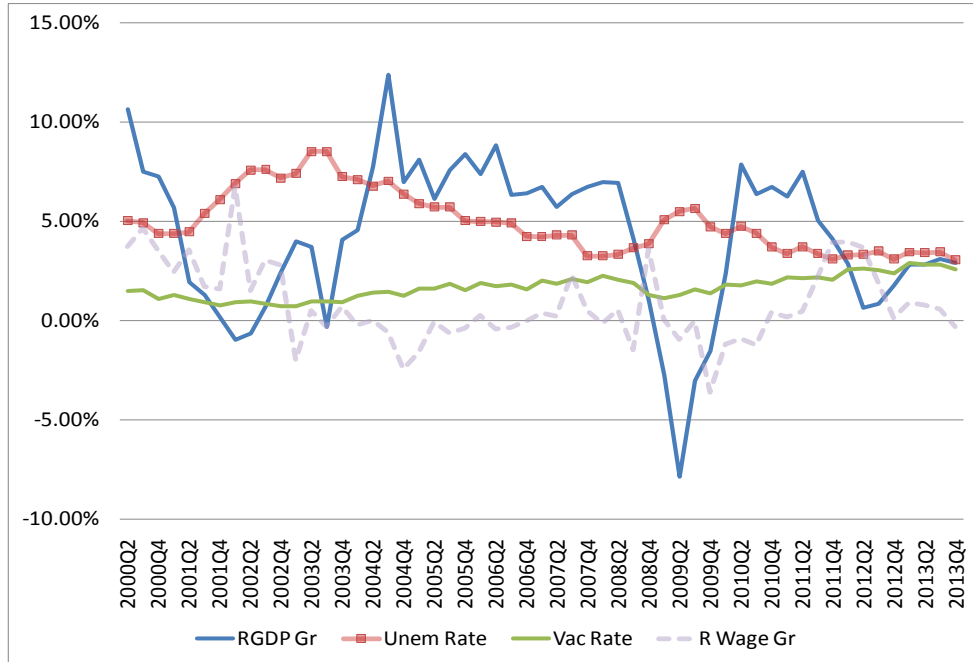
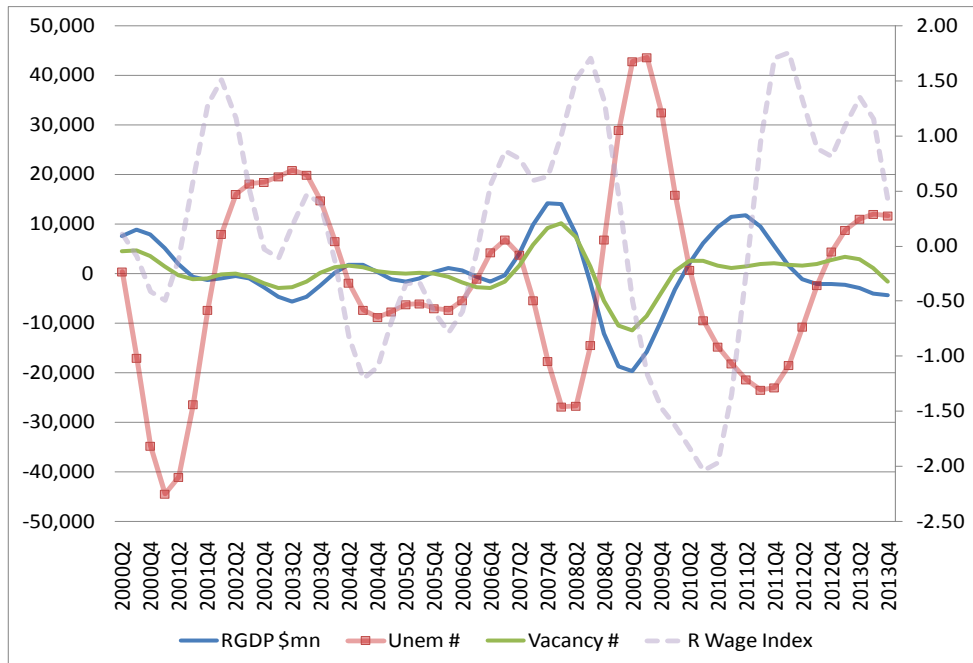


Figure 3. Filtered Data of Unemployment and Vacancy

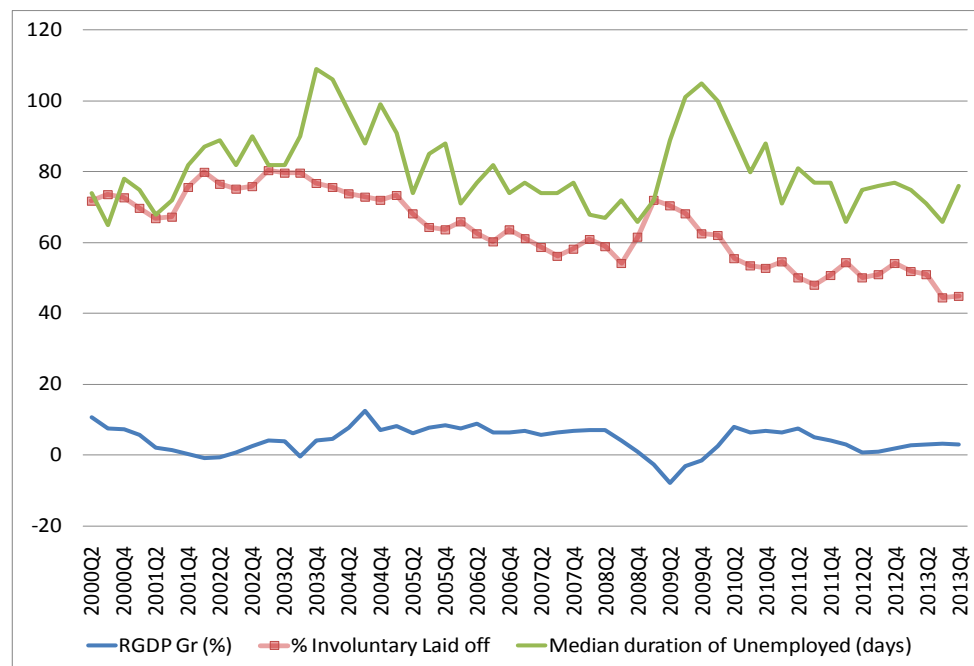


3.2. As the quarterly data are unadjusted for seasonal fluctuations and other noises, we apply the level data to the band-pass filter focusing on signals between 1.5 to 6 years. The filtered series of U , V , RGDP and the

real wage index are shown in Figure 3. Recall that these filtered series can be inferred as deviations from their respective underlying trends (hence, an indication of growth cycles). The following can be observed clearly:

- **Vacancy is pro-cyclical** and virtually synchronized with RGDP before the Subprime crisis. Their correlation is less obvious from then onwards.
- **Unemployment is counter-cyclical** and lags RGDP by about a quarter. Such relationship remains intact even after the Subprime crisis.
- Unemployment and real wages basically go in opposite directions (a negative correlation coefficient), with real wages lagging unemployment for about 1 year. For instance, real wage will peak 4 quarters after the unemployment level hit a bottom.

Figure 4. Duration of Unemployment and Risk of Layoff

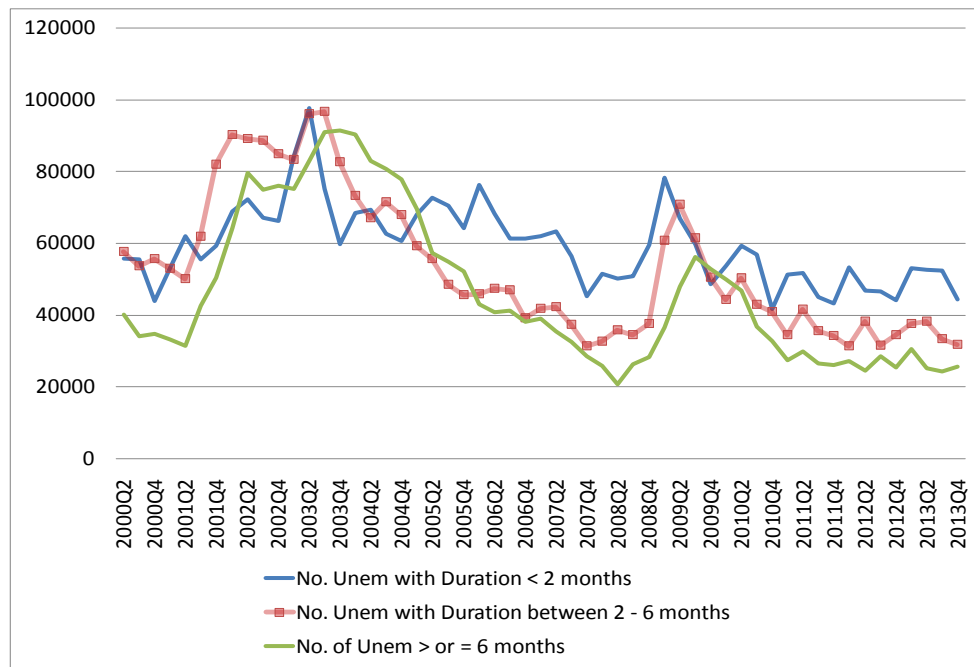


3.3. Figure 4 compares economic growth with the proportion of involuntary layoff and the duration of the unemployed. The layoff risk is higher in economic downturns and lower during expansions.

Although more volatile than the involuntary layoff percentage series, the median duration of unemployment series is also counter-cyclical.

- 3.4. Figure 5 plots the jobless population by duration of unemployment – those unemployed for less than 2 months, those unemployed for 2 months and more but less than 6, and those unemployed for 6 months and over. The three series move in the same direction in general but the longer the time of unemployment, the less volatile the corresponding series.

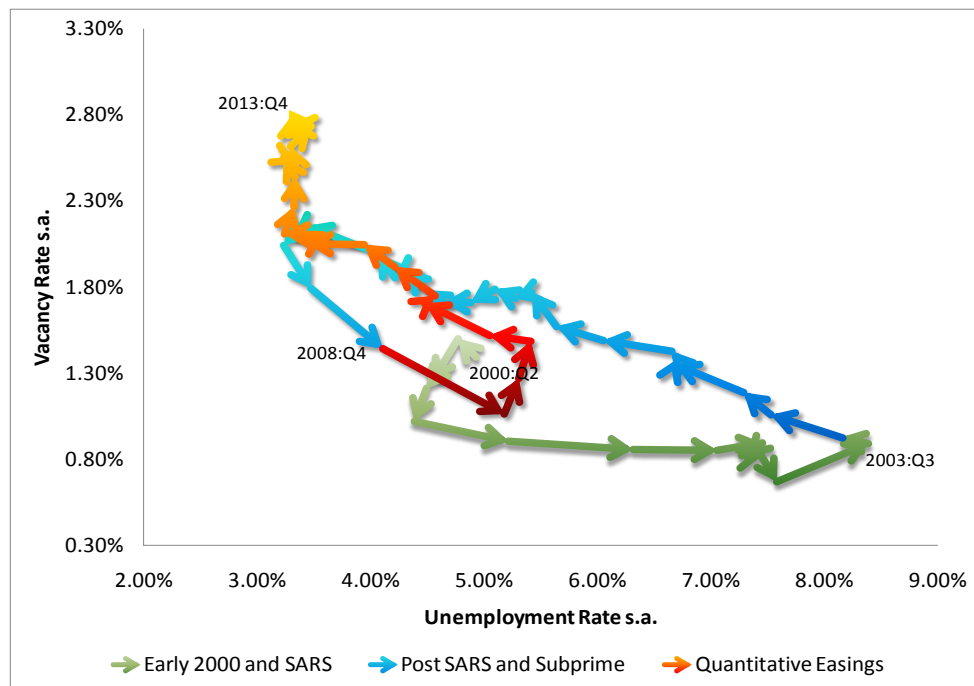
Figure 5. Distribution of Unemployment Duration



- 3.5. Finally, we plot the empirical Beveridge curve of Hong Kong in Figure 6. Note that this differs from the theoretical BC depicted by equation (3) where the change in unemployment rate per time period is zero. So, the empirical BC is the cluster of points deviating from the theoretical BC. We use arrows to show the movement of the u, v combination over time and color the curve by three different phases – the pre-SARS period, the post-SARS recovery phase, and the period from the culmination of the Subprime crisis onwards. The observation can be summed up by:

- There is a counter-clockwise looping in the points from early 2000 to late 2013, just as the matching model predicts.
- In the deflationary episode up to early 2003, the economy moves in the SE direction of the plane; and in the expansionary phase after the SARS event, the economy moves up in the NW direction. The development of the Subprime crisis forces a movement down towards SE again until the introduction of Quantitative Easing triggers another recovery.
- The last 2 years in our sample (2011:Q4 – 2013:Q4) are characterized by a near vertical movement up north in the plane. This could potentially be the result of a deterioration of matching efficiency, an increase in the willingness to hire, or a combination of both.

Figure 6. The Empirical Beveridge Curve



4. Analysis of the Matching Function

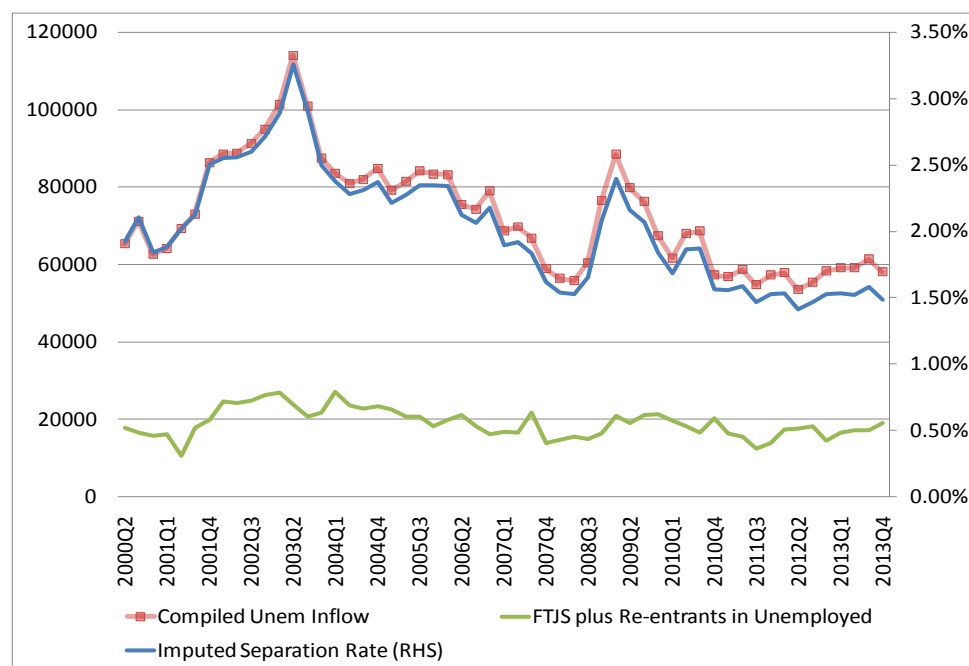
- 4.1. Our analysis involves the Beveridge curve (3) which requires, in turn, the estimation of the matching function (1). Once the parameter estimates are available, the extent of search inefficiency can be inferred and an indication of the equilibrium unemployment can be obtained.
- 4.2. Regarding the estimation, some studies focused primarily on the matching function itself, while others manipulated also information on firms hiring decision ² so the job creation structure can be simultaneously and explicitly dealt with. In this paper, we pursue the first route due to data constraint.
- 4.3. The unemployment and vacancy data of H.K. are readily available. As they come from different surveys, we rebase the vacancy data using the labor statistics in the GHS. To estimate the matching function, there is a need to find out either the number of matches $m(U, V)$ or the separation rate s both of which are not directly observable³. If the matching numbers are known, we can estimate the matching function in a straightforward manner using U and V . The situation is reminiscent of the estimation of a Cobb Douglas production function. We can either estimate an unrestricted version where U and V go into the regression equation separately with no guarantee that their powers sum to one, or we can estimate a restricted version with such a constraint imposed which requires using market tightness u/v as the only regressor. Alternatively, we can use the separation rate to estimate the BC (3) directly, again with or without the constant return to scale constraint imposed.
- 4.4. So proxies for the matches or the separation are needed. The GHS data contain the sum of first time job seekers (FTJS) and re-entrants *within*

² In some countries, micro data on the flow of job postings and vacancies are available which, together with the flow of unemployment, allows better and more complicated analysis of the matching process. An example is the work of Gregg and Petrongolo (2005).

³ Many studies in the literature used micro survey data which contain detailed information of unemployment status and duration of the unemployment spell. Obtaining the match data from those surveys is thus a headcount exercise.

those categorized as unemployed. The problem is that it excludes those first timers who managed to find a job (matched) in the reporting period and, thus, tends to understate the true number of unemployment inflow. We choose to measure the inflow using the number of those unemployed for less than 3 months⁴. Netting out from it the change in unemployment (see equation (2)) gives the number of matches for the period. The inflow proxy expressed as a ratio to labor force can serve as an estimate of the separation rate.

Figure 7. Proxy of Unemployment Inflow and Separation Rate



4.5. Figure 7 plots the compiled data used in our quantitative analysis. All the data used in model estimation and proxy compilation are seasonally adjusted with the X-12 method. To shed light on the validity of the imputed separation rates, we can compare the long term averages of different countries calculated by other researchers. Hobijn and Sahin (2007) give the estimates of s for the OECD countries. Australia has a separation rate of 1.75% (1992-2006), Canada 1.78%

⁴ We assume that those unemployed between 2 and 6 months are evenly distributed over that range. Even then, note that there is still a chance of underestimation but the downward bias is smaller than the other case.

(1992-2006), U.K. 1.53% (1992-2004), and U.S. 1.06% (2000-2006). These figures compare to an average of 2.03% we have from the imputed series.

- 4.6. We do the matching function estimation and Beveridge curve estimation separately for both the restricted and unrestricted cases. As the BC experienced various shifts over the studied time frame (see Figure 6), we choose a relatively “stable” subsample 2003:Q3 – 2008:Q1 to perform the estimation. The specific models estimated are:

$$\log H_t = \mu + \alpha \log U_t + \beta \log V_t + \varepsilon_t, \quad (4)$$

$$\log \frac{H_t}{U_t} = \mu + \beta \log \frac{V_t}{U_t} + \varepsilon_t, \quad (5)$$

$$\log(sE_t) = \mu + \alpha \log U_t + \beta \log V_t + \varepsilon_t, \quad (6)$$

$$\log\left(\frac{sE_t}{U_t}\right) = \mu + \beta \log \frac{V_t}{U_t} + \varepsilon_t, \quad (7)$$

where E_t is the employment level and $\alpha + \beta$ not restricted to 1 in models (4) and (6). The exponential of μ is the A in the matching function.

5. Empirical Results and Their Implications

- 5.1. Table 1 summarizes the results of the unrestricted models (4) and (6) and the restricted models (5) and (7). As in the case of production function estimation, restricted models usually give more desirable and intuitive results.
- 5.2. The two restricted models have both the intercept term and the estimated coefficients statistically significant. Quantitatively, the results are similar and we will base our analysis on the estimates of model (7) (last column in Table 1) as the relationship is the Beveridge curve outright.
- 5.3. Note that $e^{\hat{\mu}} = \hat{A} = 0.5586$ is the index of matching efficiency over the relatively tranquil period 2003:Q3 – 2008:Q1. In the meantime, the

model restriction means that $\hat{\beta} = 1 - \hat{\alpha} = 0.3158$ or $\hat{\alpha} = 0.6842$. So the elasticity of matching with respect to unemployment is around 0.68.

Table 1. Regression Results of Restricted and Unrestricted Models

Models	(4)	(5)	(6)	(7)
Constant return to scale restriction	No	Yes	No	Yes
Implication	Matching function	Matching function	Beveridge curve	Beveridge curve
Method	OLS	OLS	Cochrane-Orcutt	Cochrane-Orcutt
Dependent Variable	$\log H_t$	$\log(H_t/U_t)$	$\log(sE_t)$	$\log(sE_t/U_t)$
Coefficients of:				
Constant	2.3048	-0.5271*	-7.8326	-0.5823*
$\log U_t$	0.6587*	-	0.9722*	-
$\log V_t$	0.0960	-	0.6591*	-
$\log(V_t/U_t)$	-	0.2158*		0.3158*
R^2	0.6766	0.5520	0.7153	0.7126
Durbin Watson d	1.8501	1.7950	1.7751	1.5415
Error variance	0.0105	0.0100	0.0021	0.0021

Remarks: An asterisk indicates significance at the 5% level,

- 5.4. Barlevy (2011) shows how we can make use of the matching function to do simple analysis. The idea is to manipulate two equilibrium conditions. The first is the BC from (3) which can be rewritten as:

$$v = \left[\frac{s}{A} (u^{-\alpha} - u^{1-\alpha}) \right]^{1/(1-\alpha)} \quad (8)$$

and the fact that the optimal hiring policy should obey:

$$\underbrace{\frac{m(u, v)}{v}}_{\text{job filling prob.}} \times \underbrace{B}_{\text{marginal benefit of job posted}} = \underbrace{C}_{\text{marginal cost of job posted}} \quad (9)$$

or, with Cobb Douglas matching

$$A \left(\frac{u}{v} \right)^\alpha \times B = C,$$

$$\Rightarrow \frac{u}{v} = \left(\frac{C}{AB} \right)^{1/\alpha}. \quad (10)$$

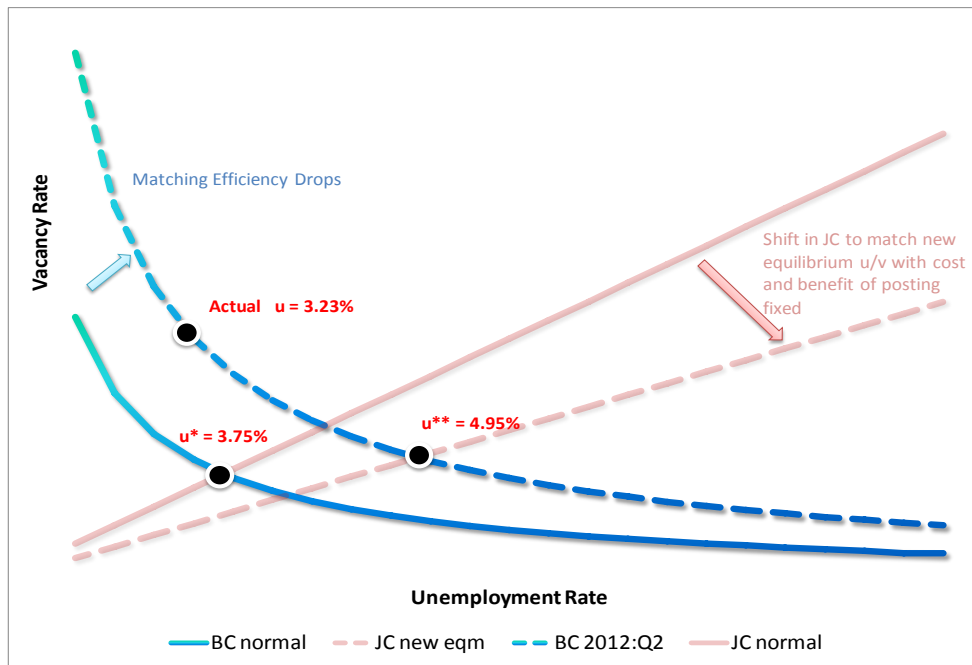
- 5.5. Should there be a shift in the BC, the unemployment, vacancy and separation information allow us to find out the new level of matching efficiency via (8). We can then compare the long run u/v ratio to that associated with the new BC and trace out the extent of shift in the job creation curve (JC) as well.
- 5.6. We illustrate this with a local example. Recall that our empirical BC is estimated using data before the impact of the financial crisis is realized and before the implementation of the minimum wage (SMW). We will look at the implication of the matching model on the unemployment situation in 2012:Q2, i.e. one year after the SMW commenced.
- 5.7. Recall that our BC is characterized by $\hat{\alpha} = 0.6842$. The 2010:Q2 unemployment and vacancy are $u = 3.23\%$ and $v = 2.53\%$ respectively. Plugging these values into equation (8) gives that value of the matching efficiency that is consistent with a shift to a new BC, if such a shift exists. The solution is $A_1 = 0.4579$ as compared to the original level $\hat{A} = 0.5586$. In other words, **matching efficiency has decreased 18% by mid-year 2012.**
- 5.8. Using the long run average unemployment rate from 1982 to 2013 as a steady state proxy ($u^* = 3.75\%$) and given the original BC estimates, we can arrive at the steady state vacancy rate of $v^* = 1.15\%$. Thus, the steady state $u/v = 3.25$. To be consistent with the job creation condition implied in (10), holding cost and benefit of posting fixed, the u/v ratio will have to increase by a factor of

$$\left(\frac{C}{A_1 B} \right)^{1/\hat{\alpha}} / \left(\frac{C}{AB} \right)^{1/\hat{\alpha}} = 1.337.$$

The new equilibrium u/v ratio should be $3.25 \times 1.337 = 4.345$. Plug this into the BC equation (3) gives an equilibrium unemployment rate of 4.95%.

5.9. The situation can be summarized by Figure 8. The BC normal corresponds to our estimated BC with an associated steady state unemployment rate of 3.75%. The data 2012:Q2 confirms a drop in the value of A (matching efficiency) resulting in a rightward shift in the BC. Assuming no change in posting cost and benefit, we can use equation (8) and (10) to find out what the new equilibrium unemployment will be. In this example, the new steady state rate is 4.95%. This, of course is higher than what we actually observed for 2012:Q2 which is a mere 3.23%. So, the actual location implies an upward movement (an economic growth) along the new BC from its equilibrium point.

Figure 8. Framing 2012:Q2 into the context of the Matching Model



5.10. What actually caused the decline in matching efficiency is not completely certain, but the introduction of SMW is undeniably a structural change that may have played a crucial role. For instance, the legislation might have improved the bargaining power of the employees and job seekers, and certain jobs may find it increasingly difficult to get the right hire. These developments would reduce matching efficiency and such deterioration could have propped up unemployment rate to as high as 4.95%, other things held constant.

However, the actual unemployment rate was just a relatively meager 3.23% which is quite a deviation from the calculated equilibrium. This could be underlined by the relative strength of the domestic economy in the post quantitative easing period. Could this state of unemployment (around 3.2%) evolve into a new equilibrium? The answer is positive, provided that the cost-benefit ratio C/B drops thereby swinging the JC in an anti-clockwise direction.

6. Conclusion

- 6.1. This paper puts local unemployment into the perspective of a labor search/matching model. Consistent with empirical evidence elsewhere, the Beveridge curve records fluctuations in the unemployment vacancy pair in a counter-clockwise pattern. Further analysis is made possible by econometric estimation of an explicit matching function.
- 6.2. An illustration of the comparative statics is provided with reference to the post-SMW labor market development. Again, the actual turnout is largely consistent with the model prediction.
- 6.3. The analysis can potentially be improved if data on hiring decisions are available. Under such a circumstance, more precise modeling and estimation of the flow of unemployment and vacancy is possible. This could shed light on whether a current state is a short term deviation from the steady state or is indeed a long run equilibrium.

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