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An equivalence scale for Hong Kong

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Abstract

This article presents estimates of an equivalence scale for Hong Kong with consumption data from the 2014/15 Household Expenditure Survey (HES). Based on these estimates, international equivalence scales such as the modified OECD scale and the square root scale appear to overstate the degree of economies of scale in local household consumption. An exponential scale with parameter 0.75 ($N^{0.75}$, where N is the number of household members) would fit the observed data more closely.

香港的相等表

摘要

本文利用 2014/15 年住戶開支統計調查的消費數據，估算香港的相等表。估算結果顯示國際上的相等表，例如經調整的經濟合作及發展組織相等表和平方根相等表，似乎會高估本地住戶消費的規模效益程度。以 0.75 為參數的指數相等表($N^{0.75}$，其中 N 為住戶人數)則和觀察到的數據較為吻合。

I. INTRODUCTION

1. A frequently-encountered problem when measuring household welfare is the need to take into account differences in household composition. Intuitively, it is clear that larger households need more resources than smaller households, and that the needs of children may be different from the needs of adults. This intuition is captured by the idea of an equivalence scale.

2. "Equivalized" measures of household welfare, such as poverty or inequality, are based on standardized household income. In other words, household income is divided by a numerical factor that depends on the number of adults and children present. A well-known example of such a scale is the "old" OECD scale, which assigns a value of 1 to the head of household, 0.7 to each additional adult, and 0.5 to each additional child.[1] Equivalized household income is then equal to household income divided by the sum of these values for the whole household (e.g., for a household of two adults, equivalized household income would be equal to household income divided by 1.7).

3. Unfortunately, no international consensus exists as to how an equivalence scale should be constructed. One well-known international report (the 1995 report on poverty measurement by the U.S. National Academy of Sciences (NAS)), after reviewing the relevant literature, found fault with each of the established methods before pragmatically conceding that "some correction is better than no correction" (p. 175).[2] Two decades later, the OECD surveyed the literature once more and concluded that "no single standard has emerged" (p. 173).[3] Nevertheless, given the importance of accounting for differences in household composition, the matter remains one of considerable interest.

4. The purpose of this article is to empirically estimate an equivalence scale with data from Hong Kong. The next section provides background information on the main types of equivalence scales that are in common use. The third section sets out details on how an equivalence scale may be estimated from Household Expenditure Survey (HES) data, and the fourth section sets out the empirical results. The fifth section concludes.

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[1] OECD. (n.d.) *What are equivalence scales?* http://www.oecd.org/eco/growth/OECD-Note-EquivalenceScales.pdf

[2] Citro, C.F., and R. T. Michael (eds). (1995) *Measuring Poverty: A New Approach*. National Academies Press: Washington, D.C. http://www.nap.edu/catalog/4759.html

[3] OECD. (2013). *OECD Framework for Statistics on the Distribution of Household Income, Consumption, and Wealth.* OECD Publishing: Paris. http://dx.doi.org/10.1787/9789264194830-en

II. TYPES OF EQUIVALENCE SCALES

5. Buhmann et al. (1988) provide a useful typology of the various methods that have been used to construct equivalence scales.[4] The four types of equivalence scales are as follows:

(a) *Expert Statistical* – devised by a panel of experts, mainly for the purpose of measuring welfare or poverty. The OECD scale is an example of this type of scale.

(b) *Expert Programme* – also devised by a panel of experts, but for the purpose of defining and calculating entitlements to social benefits.

(c) *Consumption* – based on survey data and requires a proxy for household welfare. Reflects the additional expenditure required to maintain a household's welfare at its current level, given a change in size or composition.

(d) *Subjective* – also based on survey data, but, in lieu of a proxy, aims to measure welfare directly by soliciting the respondent's own judgment (e.g., with questions as to whether or not the respondent feels the household's income is sufficient to meet its needs).

6. All equivalence scales, by their nature, involve judgments as to the varying needs of households of different types. This is most obvious in the case of expert programme scales, which directly link a household's social benefit entitlement to its size and composition. In this case, the equivalence scale simply reflects the policy maker's (or society's) judgment concerning the resources that households of different sizes and compositions ought to be provided with under the scheme concerned. Similarly, expert statistical scales reflect the views and opinions of the expert panel, though the question posed is more abstract (e.g., who should be counted as "poor").

7. As for survey-based equivalence scales, because consumption scales rely on a proxy of household welfare, judgment is involved when choosing a proxy that seems the most suitable or appropriate for this purpose. Subjective scales, for their part, rely on the judgments of the households themselves.

[4] Buhmann, B., L. Rainwater, G. Schmaus, and T. M. Smeeding. (1988). "Equivalence Scales, Well-Being, Inequality and Poverty: Sensitivity Estimates across Ten Countries Using the Luxembourg Income Study (LIS) Database." *Review of Income and Wealth*, 34(2), pp. 115-142. http://www.roiw.org/1988/115.pdf

8. The four types of scales also differ considerably in terms of the returns to scale that they expect households to realize. In particular, they can be compared in terms of their size elasticity, or the extent to which equivalized household income is reduced when household size increases. Mathematically, the size elasticity can be expressed as α in the equation

$$U = \frac{I}{S^{\alpha}}$$

where U is utility (or welfare), I is income, and S is household size. If $\alpha = 0$, then S^{α} is always equal to one and utility does not decline at all as household size increases. However, as α increases, the negative effect of household size (holding income constant) becomes more and more severe.

9. Buhmann et al. (1988) showed that size elasticities vary considerably according to the type of equivalence scale involved. In particular, subjective equivalence scales had the lowest size elasticity, followed by consumption equivalence scales, expert programme scales, and expert statistical scales (in that order). This led to a growing sense that the expert statistical scales of the 1980s put too much of a penalty on larger households.[5]

10. Eventually, and largely in response to Buhmann et al. (1988)'s work and others of its kind, Haagenars et al. (1994) proposed the "modified" OECD scale, which reduced the scale values for an additional adult and an additional child to 0.5 and 0.3, respectively (i.e., down from 0.7 and 0.5, respectively; p. 18). The size elasticity of the modified OECD scale is within the range of the expert programme scales identified by Buhmann et al. (1988). In this respect, the modification proposed by Haagenars et al. (1994) can be seen as an effort to move the OECD scale closer to the expert programme scales, which are more closely linked to governments' policy preferences.

[5] Hagenaars, A., K. de Vos and M.A. Zaidi. (1994). *Poverty Statistics in the Late 1980s: Research Based on Micro-data.* Luxembourg: Office for Official Publications of the European Communities, p. 18. http://bookshop.europa.eu/en/poverty-statistics-in-the-late-1980s-pbCA8394636/

11. The modified OECD scale remains in common use, though not by the OECD itself, which, in 2011, referred to the square root equivalence scale as the "standard OECD equivalence scale"[6] (though this remark was walked back somewhat in 2013[7]). The square root scale is simple to apply: equivalized household income is equal to household income, divided by the square root of the number of household members. No distinction is made between adults and children. The size elasticity of the square root scale is comparable to that of the modified OECD scale.

III. AN EQUIVALENCE SCALE FOR HONG KONG

III.1 Choosing a Proxy for Household Welfare

12. In Hong Kong's case, short of convening an expert panel or adding subjective questions to the Household Expenditure Survey (HES), the only type of equivalence scale that can be constructed from available data is a consumption equivalence scale. Hence, it is necessary to choose a proxy for household welfare. Because the proxy for household welfare is used to identify households with a similar standard of living, but different sizes and compositions (and, by extension, different levels of expenditure), it is the foundation of any equivalence scale estimated from objective expenditure data.[8]

13. The most common variable that is used for this purpose is the share of household expenditures allocated to "necessary" goods or services. This idea dates back to at least the 19th century, when Ernst Engel used food expenditure shares as a way to measure a family's standard of living. Engel's method was later generalized by Watts (1967), who used the share of a family's expenditures taken up by "necessities."[9] In general, it is expected that, as a family's standard of living improves, the share of expenditures taken up by necessities will decline.

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6 OECD. (2011). *Divided We Stand: Why Inequality Keeps Rising*. OECD Publishing: Paris, p. 337. http://www.rrojasdatabank.info/dividedwestand2011.pdf

7 OECD (2013), while acknowledging that "recent OECD publications" use the square root scale, also said that "no specific equivalence scale is recommended … for general use" (pp. 174-175).

8 A proxy for household welfare is implicit in any method that estimates equivalence scales from objective expenditure data. In more complex methods, however (e.g. those that require estimating complete demand systems), the proxy is not always clearly identifiable (Citro and Michael (1995), p. 174).

9 Watts, H. W. (1967). "The Iso-Prop Index: An Approach to the Determination of Differential Poverty Income Thresholds." *Journal of Human Resources* 2(1), pp. 3-18. http://files.eric.ed.gov/fulltext/ED018461.pdf

14. Exactly which goods and services are counted as "necessary" goods and services may vary according to local economic conditions. However, a key characteristic of a necessity good or service is that its expenditure share should decline as a household's standard of living improves. Moreover, whether or not a particular good or service actually has this characteristic can be objectively verified. In Hong Kong's case, the most consistent necessity goods and services appear to be "food (excluding meals bought away from home)" and "electricity, gas, and water."[10] Consequently, for the purpose of estimating an equivalence scale for Hong Kong, the household expenditure share on these items may be taken as the proxy for household welfare.

III.2 Estimating the Equivalence Scale

15. Once the proxy for household welfare has been determined, the relative expenditures of households with different sizes and compositions—but a similar standard of living—must be estimated. The relative expenditures are the basis of the equivalence scale. Typically, the relative expenditures are derived from parametric estimates of the relationship between household welfare (as measured by the proxy variable), total household expenditures, and household composition.[11] However, the validity of this approach depends critically on correctly specifying the functional relationship between these variables. For this reason, nonparametric methods—if they can be applied—are useful for exploratory purposes as they do not require this type of assumption. These methods can then inform a suitable parametric estimation strategy.

16. A suitable nonparametric estimator for this purpose is the nearest neighbour matching estimator, as defined by Abadie and Imbens (2006), who derived its large-sample statistical properties.[12] Intuitively, the estimator works by matching households with other households of different types but similar values of the household welfare variable. In this way, one can measure the average difference in expenditures between households of different types, but similar standards of living. These differences then comprise the basis of the equivalence scale.

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[10] Census and Statistics Department. (2016). "Results of the 2014/15 Household Expenditure Survey." https://www.statistics.gov.hk/pub/B71608FB2016XXXXB0100.pdf

[11] UN Food and Agriculture Organization. (2005). "Equivalence Scales: Objective Methods." http://www.fao.org/docs/up/easypol/327/equivalencescls-objective_methods_034en.pdf

[12] Abadie, A., and G. W. Imbens. (2006). "Large sample properties of matching estimators for average treatment effects." *Econometrica* 74(1), pp. 1537-1557. http://hks.harvard.edu/fs/aabadie/smep.pdf

17. In order to describe the estimator in more detail, it is necessary to introduce some additional notation. Specifically, consider a household with a fixed level of welfare. The household could be one of two types: A (the baseline for comparison) or B. If the household is type A, it spends Y_A; if it is type B, it spends Y_B. Then, within the population of interest, it is desired to estimate the average value of

$$Y_B - Y_A$$

which is the additional amount of expenditure that a type A household would require in order to maintain its standard of living as a type B household. This quantity, when expressed in relative terms (e.g., if Y_A and Y_B are measured in logs[13]), can be used to construct an equivalence scale. The statistical problem is that, for any given household, it is not possible to observe both Y_A and Y_B. It is only possible to observe one of these (i.e., the one corresponding to its actual type).

18. Abadie and Imbens' (2006) estimator solves this problem by matching each household in the data with its "nearest neighbour," which is the household of the other type that is the closest in terms of the value of the proxy variable. If the original household is a type A household, its nearest neighbour is a type B household and $Y_B - Y_A$ can be estimated as

$$Y_B^* - Y_A$$

where Y_B^* is the expenditure level of the nearest neighbour. Abadie and Imbens (2006) show that the average of $Y_B^* - Y_A$ over the sample (or $Y_B - Y_A^*$, for households that are originally type B households)[14] is a consistent estimate of $Y_B - Y_A$.

[13] Szulc, A. (2014). "Empirical versus policy equivalence scales: matching estimation." *Bank & Credit* 45(1), pp. 37-52. http://bankikredyt.nbp.pl/content/2014/01/bik_01_2014_03_art.pdf

[14] The choice of nearest neighbour is carried out with replacement, and it is not necessarily the case that there are equal numbers of households of both types in the data set. Hence, it is possible for a given household to be assigned as the nearest neighbour to more than one original household, or not to any original household.

19. When $Y_B - Y_A$ is measured in logs, it is fairly straightforward to construct an equivalence scale. In this case, $\exp(Y_B - Y_A)$ is the equivalence scale factor of household type B, relative to household type A. For example, if $Y_B - Y_A$ is equal to 0.3, it means that, on average, a type B household needs to spend 35% more than a type A household in order to maintain the same standard of living (because exp(0.3) = 1.35). Or, put another way, the income of a type B household should be divided by 1.35 in order to make it comparable to the income of a type A household. The scales for other types of households can then be estimated in the same way.

20. Once the nonparametric results are available, an appropriate functional form for the relationship between household expenditures, household composition, and the proxy for household welfare can be determined. In Hong Kong's case, the following functional form appears the most suitable for estimating the equivalence scale, in the sense that it produces results that are in line with the nonparametric estimates:

$$\ln(HHEXP_i) = \beta_0 + \beta_1 NSHARE_i + \beta_2 ADULT2_i + \beta_3 ADULT3_i + \beta_4 ADULT4_i + \beta_5 CHILD1_i + \beta_6 CHILD2_i$$

where $HHEXP_i$ is the total household expenditures of household i; $NSHARE_i$ is the ratio of expenditures on "food (excluding meals bought away from home)" and "electricity, gas, and water" to total household expenditures; $ADULT2_i$, $ADULT3_i$, and $ADULT4_i$ are binary variables equal to one if there are at least 2, 3, or 4 adults present in the household, respectively, and zero otherwise; and $CHILD1_i$ and $CHILD2_i$ are binary variables equal to one if there are at least 1 or 2 children present in the household, respectively, and zero otherwise. Households with more than 4 adults or more than 2 children are excluded because there were not enough of these types of households in the data to reliably estimate the relevant parameters.

21. Lastly, when constructing these estimates, all housing expenditures are measured according to the private rental market. Specifically, for owner-occupiers and PRH tenants, housing expenditures are equal to the net imputed rent (i.e., the imputed rent exclusive of rates, management fees, and other charges) plus other housing costs. Further, the effects of the Government's one-off relief measures are excluded (i.e., expenditures should be measured according to the market price of the goods or services in question, without subtracting any subsidy from the Government).

III.3 Simplifying the Equivalence Scale

22. For comparison purposes, it is desirable to express the equivalence scale in more familiar terms. For instance, the equivalence scale may be approximated by a two parameter "OECD-style" scale, where the first parameter is the value associated with adding an additional adult to the household (from a base of 1 for a single person household) and the second parameter is the value associated with adding an additional child. More formally, the equivalence scale factors for the OECD-style scale may be expressed as

$$1 + \theta_A \mathrm{A} + \theta_C \mathrm{C}$$

where A is the number of adults in the household (excluding the household head; and defined as those aged 15 and higher) and C is the number of children (under age 15) in the household. The parameters θ_A and θ_C can then be estimated with a linear regression model.[15]

23. An alternative is to express the equivalence scale as an exponential scale of the form

$$N^{\alpha}$$

where N is the number of household members and α is a parameter to be estimated by nonlinear least squares. The advantage of the exponential equivalence scale is that it models returns to scale in a somewhat more realistic fashion than the OECD-style scale, particularly for larger households. The square root scale is a special case of the exponential scale where α is set equal to 0.5.

24. Lastly, a third option is to use express the equivalence scale in the form recommended by the U.S. National Academy of Sciences (1995, p. 176):

$$(1 + \mathrm{A} + \gamma \mathrm{C})^{\alpha}$$

where A and C have the same definition as in the OECD-style scale and α and γ are parameters to be estimated by nonlinear least squares. The NAS-style scale essentially combines the attributes of the OECD-style scale and the exponential scale.

[15] In the regression model, the equivalence scale factor from the model in para. 20 (rescaled, if necessary, so that it is equal to 1 for single person households) is the dependent variable, and the numbers of additional adults and children in the household (i.e., not counting the household head) are the explanatory variables. Further, the constant term is set equal to 1 in order to ensure that the regression-based equivalence scale factor remains equal to 1 for single person households.

IV. EMPIRICAL RESULTS

25. The estimates of the equivalence scale from the full 2014/15 HES data set, expressed in terms of the three options described earlier (the OECD-style scale, the exponential scale, and the NAS-style scale), are set out in ***Table 1***.

Table 1: Estimates of the Equivalence Scale

Option	Equivalence Factor
OECD-style scale	$1 + 0.6318 * \mathrm{A} + 0.5196 * \mathrm{C}$
Exponential scale	$\mathrm{N}^{0.7445}$
NAS-style scale	$(1 + \mathrm{A} + 0.8521 * \mathrm{C})^{0.7575}$

26. In this table, "A" refers to the number of adults in the household (age 15 and higher) apart from the household head, and "C" refers to the number of children in the household (under age 15). "N" refers to the number of household members. The equivalence factors generated by each of the options are highly correlated ($\rho > 0.99$) with the equivalence factors generated by the non-simplified scale in para. 20.

27. The coefficient of the OECD-style scale for adults is somewhat in between that of the modified OECD scale (0.5) and the old OECD scale (0.7). However, as the modification of the OECD scale was somewhat *ad hoc* in nature (as described in para. 10), the estimated coefficient of 0.6318 may be regarded as within the range of reasonable estimates. The estimated coefficient of 0.5196 for children is close to that of the old OECD scale (0.5) and higher than that of the modified OECD scale (0.3). This is consistent with Wong et al.'s (2012) finding that Hong Kong families tend to spend a relatively large amount on children.[16]

28. The ratio of expenditures on children to expenditures on adults according to the OECD-style scale is also within expectations. In particular, the ratio of marginal expenditures on children to marginal expenditures on adults, which is 82.2% $\left(\frac{0.5196}{0.6318}\right)$, is similar to what this ratio should be according to the basic needs approach. In particular, according to Hong Kong's 1996 Basic Needs budget, the ratio of marginal expenditures on children to marginal expenditures on adults is about 78.1% $\left(\frac{HK\$1168}{HK\$1496}\right)$,

[16] Wong, C., C. Law, C. C. Ho, S. Wong, and A. M. Choy. 2012. "Feasibility Study of Low-Income Working Family Subsidy." Mimeo, Oxfam Hong Kong. https://web.swk.cuhk.edu.hk/~ckwong/Full%20report_eng_22%20june%202012.pdf

which is quite close to 82.2%.[17] The ratio from the NAS-style scale (0.8521) is also within this range, though it is not fully comparable to the other ratios because of economies of scale (as, unlike a linear scale, it requires that the original household size be held constant when comparing the marginal effects of adults and children).

29. The returns to scale parameters (0.7445 for the exponential scale, and 0.7575 for the NAS-style scale) are also within expectations. Specifically, for the U.S., the 1995 NAS report recommends a scale parameter between 0.65 and 0.75 (p. 59). In Hong Kong's case, the scale parameter appears to be at the upper end of this range. Nevertheless, the estimates are quite reasonable, as the NAS Panel's own research[18] in fact suggested a scale parameter of 0.76. Ultimately, however, the Panel chose to recommend a compromise between its own findings and the values suggested by other equivalence scales that were in use at the time.

V. CONCLUSION

30. The idea that households are able to realize economies of scale in consumption is natural and intuitive. Yet, the extent to which international equivalence scales, such as the modified OECD scale or the square root scale, are applicable to Hong Kong is largely unknown. Estimates from the 2014/15 Household Expenditure Survey (HES) indicate that the modified OECD scale and the square root scale may overstate the returns to scale that are available to households in Hong Kong. In Hong Kong's case, and because the returns to scale are clearly nonlinear, a simple exponential scale of $N^{0.75}$ or an NAS-style scale of $(1 + A + 0.85 * C)^{0.75}$ may both be useful for this purpose[19]. However, as the scale remains experimental in nature, it would be important to confirm the stability of the parameter estimates with more recent expenditure data before applying the results.

[17] Legislative Council Secretariat, Research and Library Services Division. 2005. "Benchmarks for Granting Subsidies or Financial Assistance to People in Need in Hong Kong." Appendix VII. http://www.legco.gov.hk/yr04-05/english/sec/library/0405rp07e.pdf

[18] Betson, D., and R. Michael. 1993. "A Recommendation for the Construction of Equivalence Scales." Unpublished memorandum prepared for the Panel on Poverty and Family Assistance, Committee on National Statistics, National Research Council. Cited by NAS (1995), p. 178.

[19] The parameters in the preceding scales are rounded to the nearest 0.05 for simplicity. In the NAS-style scale, "A" refers to the number of adults in the household (age 15 and higher) apart from the household head, and "C" refers to the number of children in the household (under age 15). In the exponential scale, "N" refers to the number of household members.