A State Space Model of NAIRU

Dr. William Chow

16 March 2011

Executive Summary

- This paper presents a state space model that allows for simultaneous determination of potential output and the NAIRU, both of which are unobservable by definition. **The theoretical construct of the model is the expectation augmented Phillips curve and the Okun's Law**. The potential output and the NAIRU are assumed to evolve according to random walks.

- We adopt a Bayesian approach in estimating the model. Several information criteria are used to help select a model with desirable model dimension (the amount of parameters contained in the model). Including more lagged terms of endogenous and exogenous variables enlarges the information set of expectation formation concerning inflation and produces smoother NAIRU estimates. The selected model outperforms models using HP filtered potential output.

- **The selected model generates nice mirror opposite between the output gap (the difference between log real GDP and log potential output) and the unemployment gap (the difference between actual unemployment rate and the NAIRU)**. The latter is also the cyclical unemployment.

- According to the Phillips curve, **a 1% point increase in the unemployment gap results in a contemporaneous decline in inflation rate of 0.8% point**.

- **The correlation coefficient of the output gap and the unemployment gap from 1988 to 2010 is -0.93**.

Summary Statistics of Estimated NAIRU

	Potential Output (log)	NAIRU (%)	Output Gap (%)	Unemployment Gap (%)
Full Sample				
Mean	12.57	3.88	0.036	0.022
Std. Deviation	0.26	0.89	1.28	0.023
1988-1997:3Q				
Mean		3.02		-1.07
1997:4Q-2004				
Mean		4.74		1.27

1. Introduction

This paper explores the feasibility of estimating the NAIRU of Hong Kong with a view to deliver a series that is consistent with economic theory. As is well known, the NAIRU is closely related to the structural unemployment, or the unemployment that remains once cyclical and frictional unemployment are taken out. However, an inherent predicament is that the structural unemployment is purely conceptual and totally unobservable. This means that both the *ex ante* estimation and the *ex post* sensitivity checks are going to be severely hindered by this anonymity.

Common approaches to estimate/measure structural unemployment include (i) manipulating the Phillips curve, (ii) using *ad hoc* structural models like the Beveridge curve, and (iii) resorting to time series filtering methods such as the Hodrick Prescott (HP) Filter. The model considered in this study tries to overcome drawbacks of these other approaches[1] in that we programmed robust searches to give results which are consistent with theory. In particular, the model consists of a system of equations that represent the (expectation augmented) Phillips curve, the Okun's Law, and the evolution of the potential output and the NAIRU. Estimation is performed using Bayesian methodology and Markov Chain Monte Carlo (MCMC) simulations.

[1] HP filtered series, for instance, have no economic interpretation per se; and estimated Phillips curve, if not correctly structured, may not pick up the desired signs or be consistent with other economic aggregates.

2. The Model

Like many other studies in the literature, the focal point of the model used here is the Phillips curve. However, unlike the OECD model (see Turner *et. al*, 2001), we incorporate an extra set of restriction – namely, the Okun's Law – to ensure that there are no counter-intuitive movements in the NAIRU (against movements in Real GDP). The model is in the spirit of Apel and Jansson (1999), but differs in two respects: (i) There is a lag term of output gap in our model which is non-existent in theirs, and (ii) there is no equation governing the dynamics of cyclical unemployment as in their model. The first point allows for heteroskedasticity in the Okun's Law equation[2]. The second modification eliminates potential inconsistency in the dynamics of the variables[3]. The model is formally represented by the following four equations:

$$\pi_t = \sum_{i=1}^{m} \alpha_i \pi_{t-i} + \sum_{j=1}^{2} \sum_{i=0}^{n} \varphi_{ji} z_{jt-i} + \sum_{i=0}^{k} \beta_i (u_{t-i} - u_{t-i}^*) + \varepsilon_t^{\pi} \qquad (1)$$

$$y_t - y_t^* = \phi \Delta y_t^w + \gamma(y_{t-1} - y_{t-1}^*) + \sum_{i=0}^{p} \delta_i (u_{t-i} - u_{t-i}^*) + \varepsilon_t^o \qquad (2)$$

$$u_t^* = u_{t-i}^* + \varepsilon_t^u \qquad (3)$$

$$y_t^* = \tau + y_{t-1}^* + \varepsilon_t^y \qquad (4)$$

where

- The equation (1) and (2) are the expectation augmented Phillips curve and the Okun's Law, respectively. (3) and (4) describe how the unobserved NAIRU and potential output evolve.
- π_t, y_t, and u_t are inflation rate, log RGDP (deseasonalized) and unemployment rate, respectively.
- An asterisk denotes the unobserved "natural" level. u_t^* is thus the NAIRU and y_t^* the potential output at time *t*.
- The lagged values of inflation are used as a proxy for expected inflation. z_{jt} is a set of variables that capture supply shocks and are modeled with, as in other studies, changes in oil prices $(j = 1)$ and changes in import prices $(j = 2)$.

[2] The estimates of NAIRU and potential output are much better with the lag term is included, and theoretically makes more sense due to persistence in output data.

[3] In the special case of NAIRU following unit root, such a restriction in cyclical unemployment will confound with equation (3).

- The variable Δy_t^w measures external shocks and is proxied by the change in US RGDP.
- ε_t^π and ε_t^o are random error terms of equations with observable data, and ε_t^u and ε_t^y are random errors of equations with unobservable state variables u_t^* and y_t^*.
- τ is a trend term to be estimated. All other Greek letters are parameters/coefficients to be estimated. For simplicity, the lag lengths $k = p$ are assumed.

The model can be restated more compactly as:

$$Y_t = AZ_t + B\theta_t + \zeta_t \qquad (5)$$

$$\theta_t = C + D\theta_{t-1} + E\eta_t \qquad (6)$$

where $\zeta_t \sim N(0_{2\times 2}, \Omega)$ and $\eta_t \sim N(0_{2\times 2}, \Psi)$ are mutually uncorrelated Gaussian vectors. The state-space model (5) and (6) is a rather common formulation in empirical economics. The first equation is the measurement equation (because all measurable variables enter here) and the second the transition equation (because it describes how the unobservable variables θ evolve over time). There are standard estimation methods to solve the model but the performance will depend crucially on whether misspecification is severe. In our context, since both B and θ are unobserved, to be able to correctly identify them jointly will indeed be very difficult[4].

We adopt a Bayesian approach in this paper to explore the complex admissible parameter space. The model is solved using Markov Chain Monte Carlo methods, details of which are in the Appendix.

3. Empirical Results

3.1 Data

The Hong Kong data used are official figures from the C&SD website and the output data are seasonally adjusted. The oil prices are available from U.S. Energy Information Administration while the US RGDP figures are from the US Bureau of Economic Analysis. The data frequency is quarterly and the sample runs from 1987:Q1 to 2010:Q4.

3.2 Model Selection

The model order, i.e., the choice of parameters m, n, k and p, is determined by various selection criteria – the Log-likelihood, the Akaike Information Criterion (AIC) and the

[4] In simple regression models, θ is just another observed variable and estimating B will be straightforward.

Bayesian Information Criterion (BIC). All three gauge model performance in accordance with the maximum likelihood principle, except that the latter two include a penalty element to discriminate against overfitting. Table 1 and Figure 1 highlight the findings. Figure 1 plots the actual unemployment rate and the estimated NAIRU based on different model configurations, and Table 1 gives the summary of the selection criteria. In general, the selection criteria favor parsimonious models (models with a small dimension of parameters) as expected, but these "smaller" models tend to generate slightly more volatile estimates of NAIRU that track more closely the actual unemployment rates. This is fine from a statistical point of view, but it would mean that people who are structurally unemployed could react quickly to economic conditions and leave/re-enter the labor market in an efficient way. While this is not totally impossible, a smoother NAIRU seems to be a more plausible conjecture. In addition, a larger m and n implies that people use a larger information set to help them predict inflation. On considering all these, we chose the model highlighted in Table 1 where $m, n = 4$ and $k, p = 3$.

Table 1: Model Selection Summary

Model		Log-Likelihood	AIC	BIC
$m = n = 1,\ k = p = 0$		477.02	-918.03	-872.06
$m = n = 1,\ k = p = 1$		461.54	-883.09	-832.01
$m = n = 2,\ k = p = 0$		445.53	-849.06	-795.65
$m = n = 2,\ k = p = 1$		444.28	-842.55	-784.06
$m = n = 2,\ k = p = 2$		450.50	-851.00	-787.42
$m = n = 3,\ k = p = 0$		452.05	-856.11	-795.33
$m = n = 3,\ k = p = 1$		424.19	-796.38	-730.54
$m = n = 3,\ k = p = 2$		430.36	-804.72	-733.80
$m = n = 3,\ k = p = 3$		435.44	-810.89	-734.91
$m = n = 4,\ k = p = 0$		431.80	-809.60	-741.52
$m = n = 4,\ k = p = 1$		422.58	-787.16	-714.52
$m = n = 4,\ k = p = 2$		457.31	-852.61	-774.44
$m = n = 4,\ k = p = 3$		436.00	-806.00	-722.78
$m = n = 4,\ k = p = 4$		430.63	-791.26	-703.00
$m = n = 4,\ k = p = 3$	**HP- Trends**	324.05	-622.45	-539.23

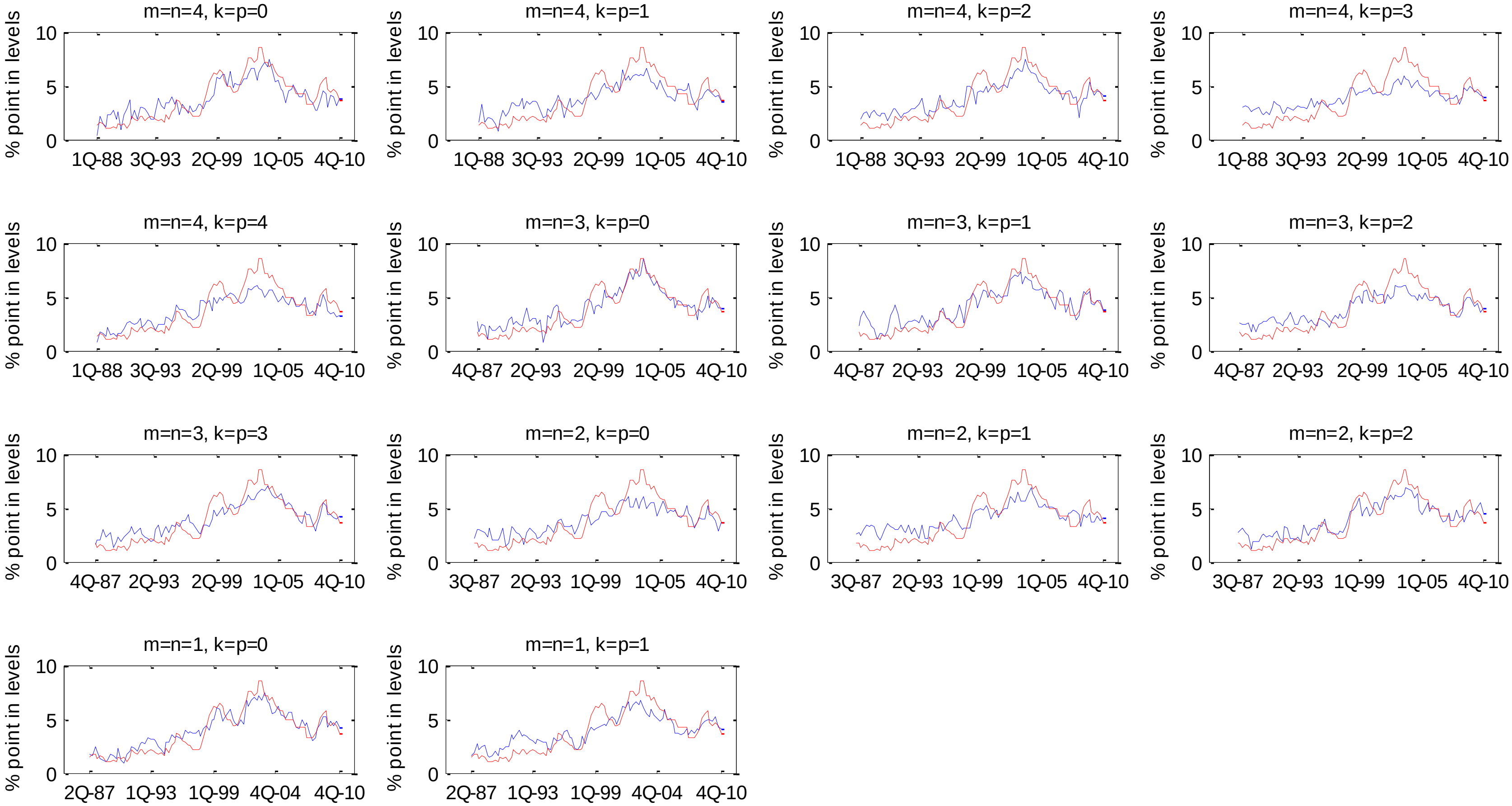
Remark: The selection criteria are to maximize Log-likelihood and to minimize AIC and BIC.

Table 2: MCMC Estimates of Selected Model Parameters

Parameters	Posterior Estimates	95% HPD Interval	Significance
Phillips Curve			
α_1	0.7507	(0.5934, 0.9086)	*
β_0	-0.8005	(-1.1596, -0.4446)	*
β_1	0.1552	(-0.3421, 0.6613)	
Okun's Law			
ϕ	0.0001	(-0.0003, 0.0005)	
γ	0.4203	(0.2632, 0.5763)	*
δ_0	-0.0214	(-0.0286, -0.0142)	*
δ_1	0.0101	(0.0005, 0.0197)	*
Others			
τ	0.0274	(0.0017, 0.0490)	*

Remark: The HPD (Highest Probability Density) intervals are Bayesian version of confidence intervals, and asterisks indicate significance at the 95% level.

Figure 1: Estimated NAIRU Based on Different Model Configurations



3.3 Model Estimates

In line with the prediction of economic theory, β_0 in the Phillips curve is negative and significant. Other thing being the same, **a 1% point increase in the cyclical unemployment (or the unemployment gap, $u - u^*$) results in a contemporaneous decline in inflation rate of 0.8% point**. Meanwhile, α_1 is about 0.75 implying that there is strong lingering effect of past inflation. This also means that current inflation is a good proxy of expected inflation.

The model also picks the correct sign for the parameters in the Okun's Law. δ_0 is negative and significant implying that **the output gap and cyclical unemployment are negatively related contemporarily. A 1% point increase in cyclical unemployment is consistent with a fall in RGDP (with respect to potential output) of about 2.14%.** There is a fair amount of autocorrelation in the output gap, as witnessed by the value of γ. Once lagged output gap is taken into account, foreign output shock has little explanatory power in the equation. Finally, τ equals to 0.0274, which suggests that **the intrinsic growth rate of potential output is about 2.7% per month in the absence of shocks**.

The following diagrams show our estimation results. Figure 2 plots the actual RGDP and the potential output. **From 1988-1991 when inflation was relatively high, actual RGDP hovered at an average of 2.5% above the potential output. The overshooting continued for more or less a decade until the trend reversed. Except for the brief recovery in 2000-2001, the output gap stayed negative all the way up to 2005, nudged back up to positive region from 2006 to early 2008 before seeing another dip in the financial tsunami era**.

Figure 3 shows the actual unemployment rate and the estimated NAIRU. The random walk transition of the NAIRU specified in the model might have resulted in some bumpiness observed, and a 3-quarter moving average is plotted for easy reference. **In general, the unemployment gap exhibits a clear mirror opposite of the output gap**.

Figures 4 to 6 indicate the situations for the Phillips curve, the Okun's Law, and the relationship between nominal wages and the cyclical unemployment, respectively. Again, **the estimated model asserts that (i) inflation is negatively related to cyclical unemployment, (ii) output gap is negatively related to cyclical unemployment, and (iii) wage pressure diminishes with higher cyclical unemployment – when the unemployment gap is 0, the annual growth in nominal wages is close to** . All these evidence imply that the NAIRU estimated is in concord with economic theory.

Figure 2: Potential Output from the Estimated Model

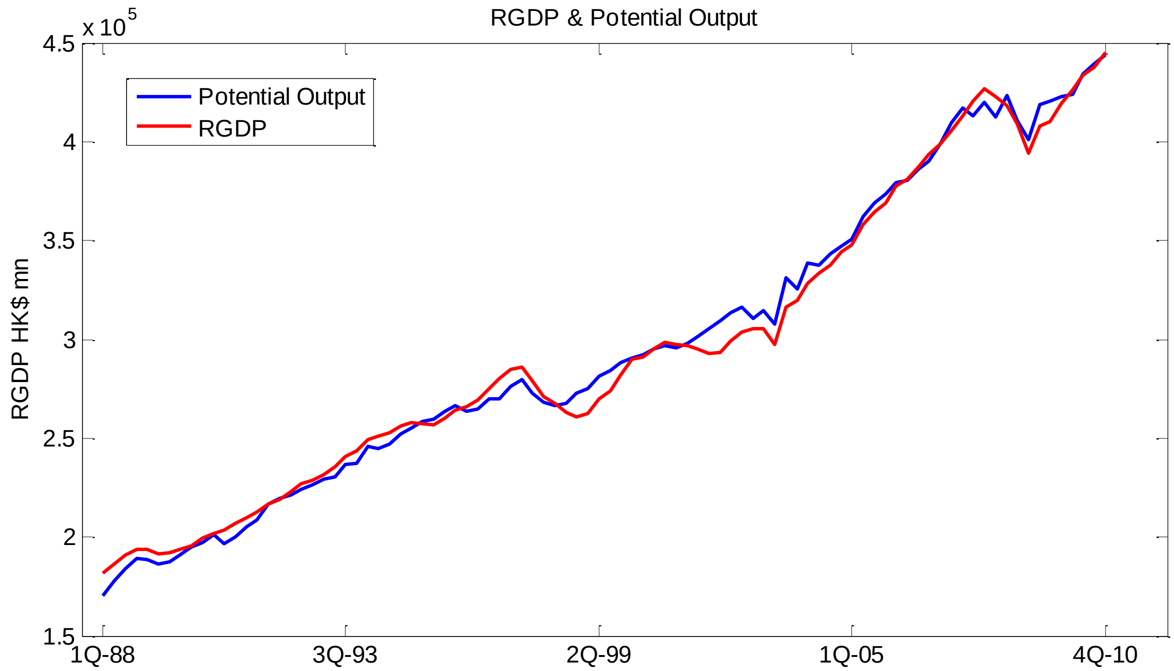


Figure 3: NAIRU from the Estimated Model

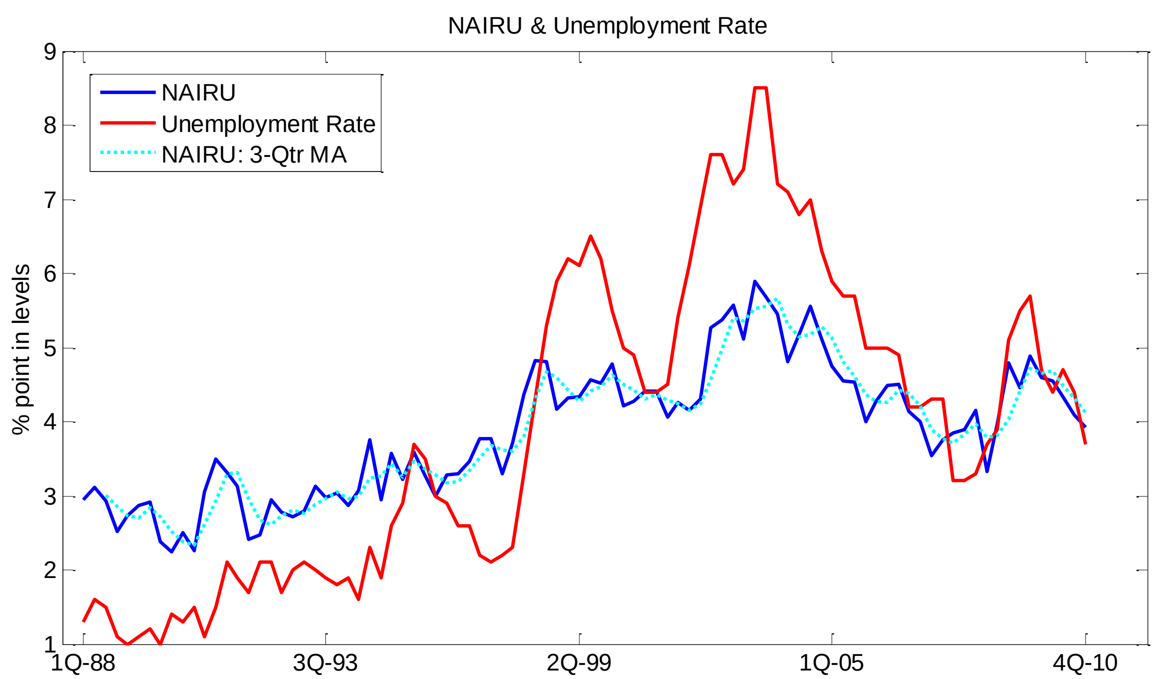


Figure 4: The Phillips Curve

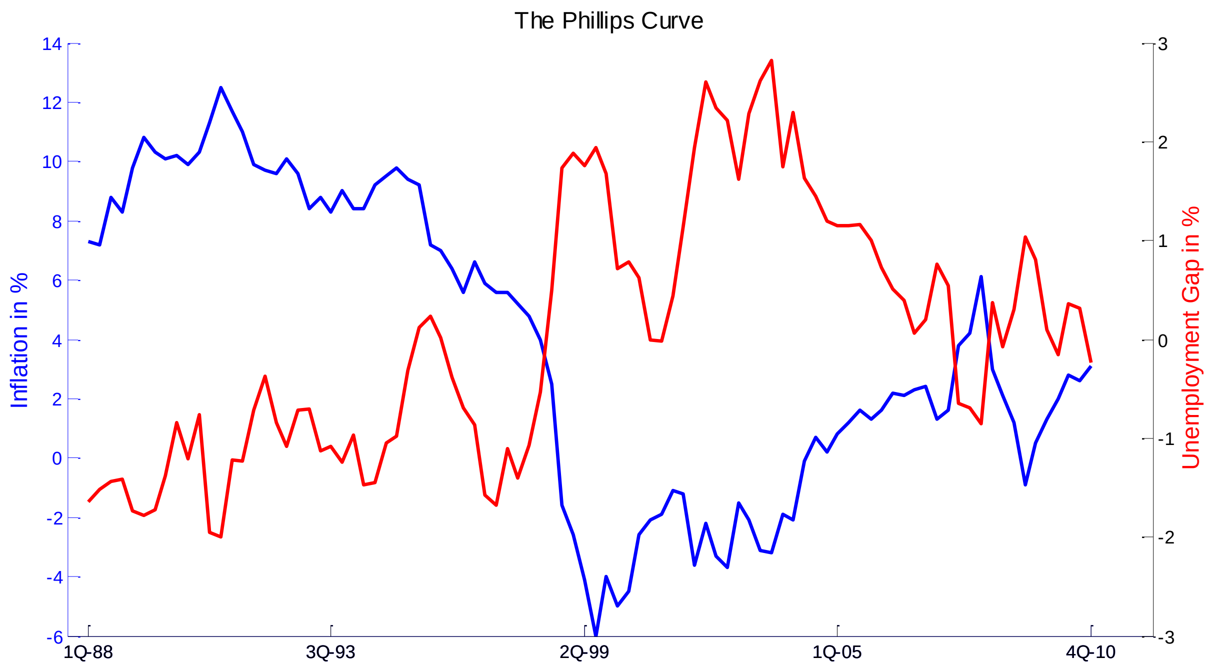


Figure 5: The Okun's Law

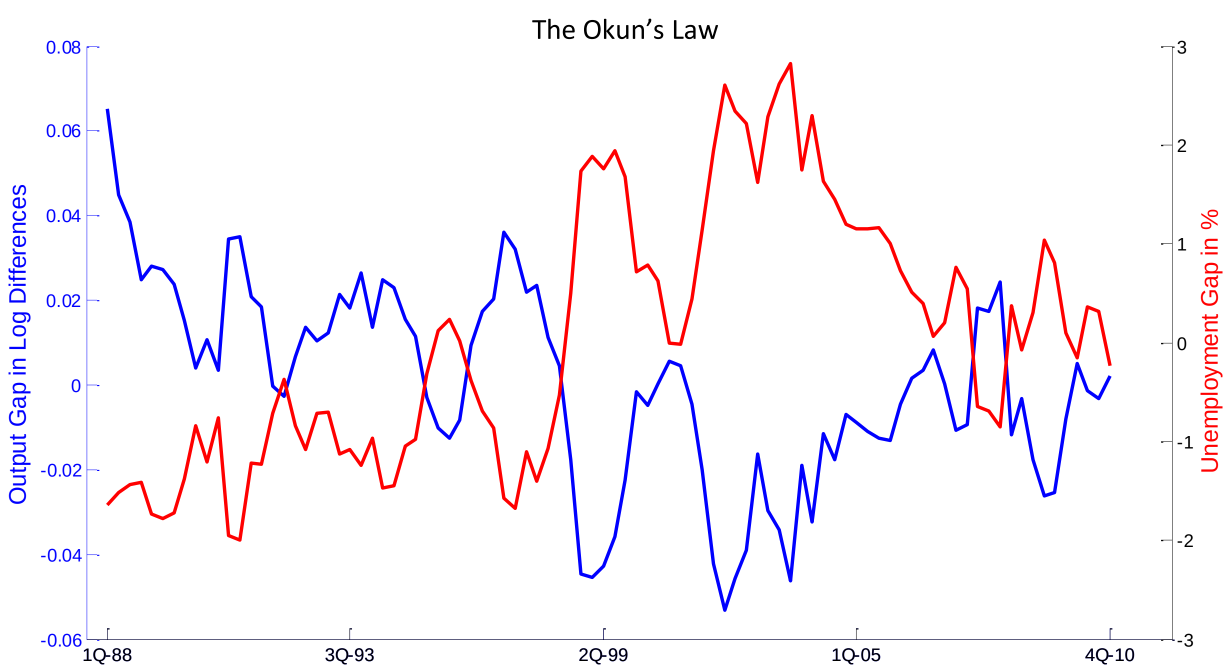
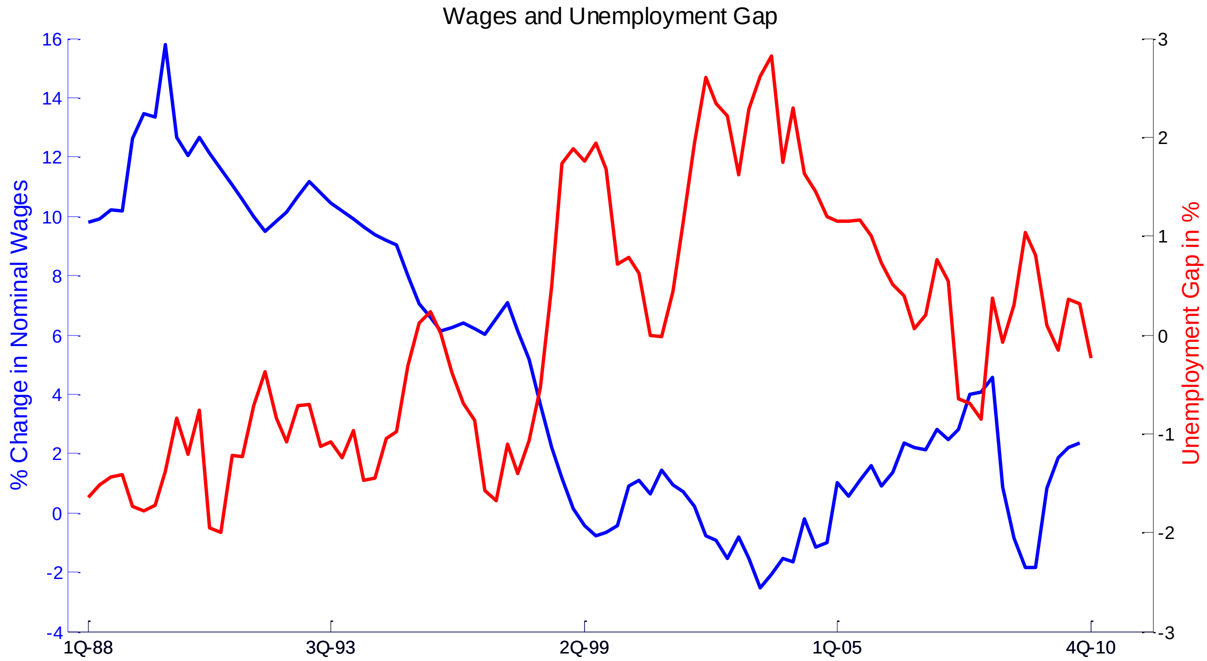


Figure 6: Nominal Wage and the Unemployment Gap

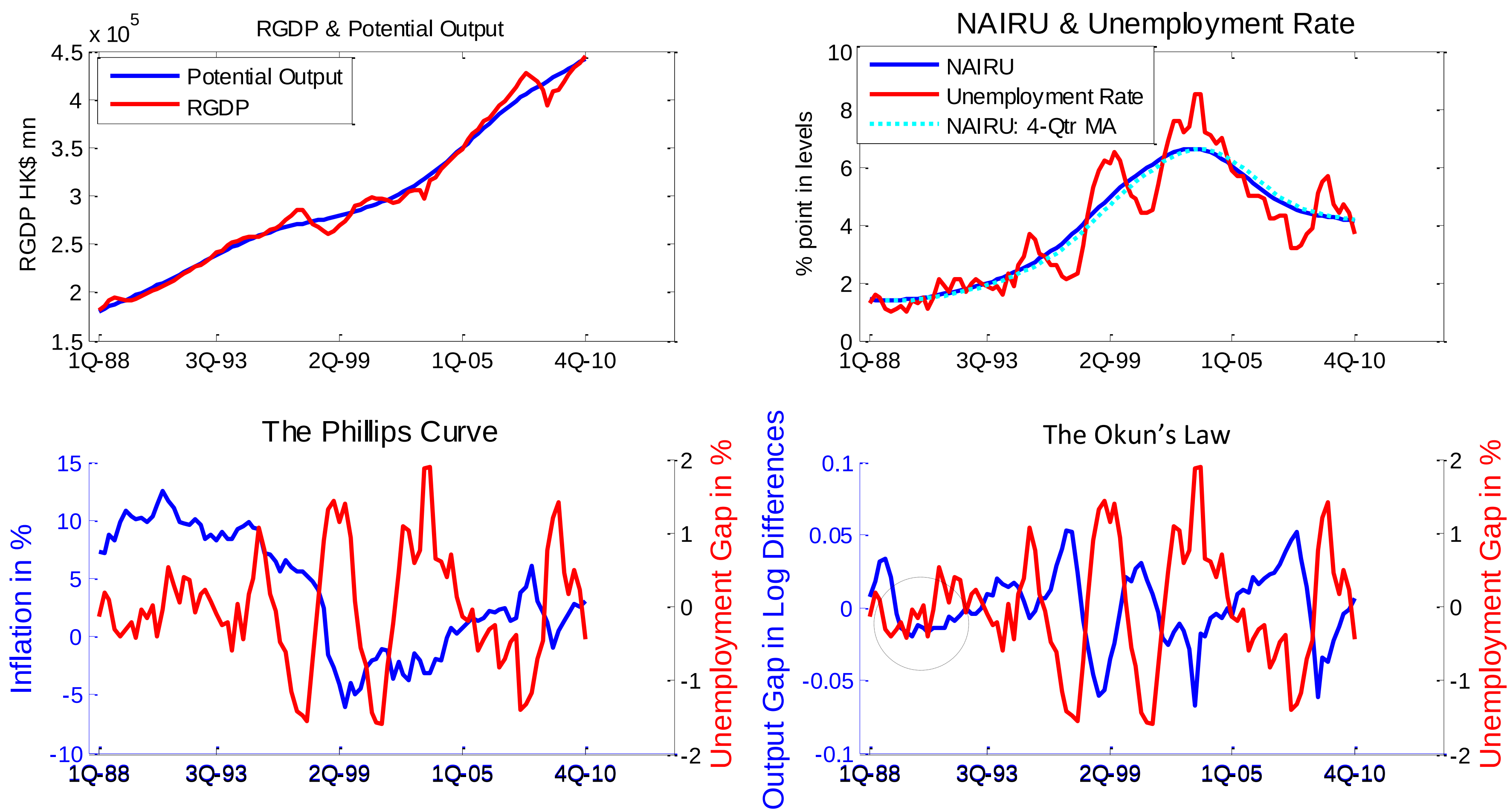


3.4 Model Comparison

In academic works of Bayesian statistical analysis, model comparison is mostly dealt with using the Bayes Factor[5] which is highly computation intensive. We opt for the more convenient information criteria discussed in section 3.2, and the figures can be found in the highlighted rows of Table 1. The benchmark model using HP filtered trend as potential output shares the same dimension as the selected model. It can be seen that the selected model beats the benchmark on all counts (having a larger log-likelihood and a smaller AIC and BIC). The generated series of the benchmark models are illustrated in Figure 7.Tthe predicted Okun's Law of the HP benchmark model indicates that the output gap in the late '80s and early '90s was negative when the economy was experiencing double digit inflation. So from both the statistical measures and the data patterns observed, the selected model offers an estimate of NAIRU that looks much more consistent with fundamental economic theory.

[5] This is pretty much a kind of likelihood ratio. See for instance Gelman et al. (2000) for details.

Figure 7: Prediction Using HP Filtered Trends for Potential Output and NAIRU



4. Reference

Apel, M. and P. Jansson, 1999. *Systems Estimates of Potential Output and the NAIRU*, Empirical Economics, v24 n3.

Ball, L. and N.G. Mankiw, 2002. *The NAIRU in Theory and Practice*, Journal of Economic Perspectives, v16 n4.

Benes, J. and P. N'Diaye, 2004. *A Multivariate Filter for Measuring Potential Output and the NAIRU: Application to the Czech Republic*, IMF Working Paper 04/45.

Gelman, A., Carlin, J.B., Stern, H.S. and D.B. Rubin, 2000. *Bayesian Data Analysis*. First Reprint, Chapman & Hall/CRC, Boca Raton, FL..

Gerlach-Kristen, P., 2004. *Estimating the Natural Rate of Unemployment in Hong Kong*, University of Hong Kong, HIEBS Working Paper 1085.

Geweke, J. and H. Tanizaki, 2001. *Bayesian Estimation of State-space Models using the Metropolis-Hastings Algorithm within Gibbs Sampling.* Computational Statistics & Data Analysis, v37 n2.

Turner, D., L. Boone, C. Giorno, M. Meacci, D. Rae and P. Richardson, 2001. *Estimating the Structural Rate of Unemployment for the OECD Countries*, OECD Economic Studies No.33.

5. Appendix

5.1 The reparameterized model (5) – (6) have components defined as follows:

$$Y_t = \begin{bmatrix} \pi_t \\ y_t \end{bmatrix}, \qquad \theta_t = [u_t^*, \quad \dots \quad u_{t-k}^*, \; y_t^*, y_{t-1}^*]' \;,$$

$$Z_t = [\pi_{t-1}, \quad \dots \quad \pi_{t-m}, \; z_{1t}, \quad \dots \quad z_{2t-n}, \; u_t \quad \dots \quad u_{t-k}, \; y_{t-1}, \Delta y_t^w]',$$

$$A_0 = \begin{bmatrix} \alpha_1, & \dots & \alpha_m, \; \varphi_{10}, & \dots & \varphi_{2n} \\ & & 0_{1\times(m+2n+2)} & & \end{bmatrix}, \; B_0 = \begin{bmatrix} -\beta_0, & \dots & -\beta_k \\ -\delta_0, & \dots & -\delta_k \end{bmatrix},$$

$$A = \begin{bmatrix} A_0 & -B_0 & \begin{matrix} 0 & 0 \\ \gamma & \phi \end{matrix} \end{bmatrix}, \; B = \begin{bmatrix} B_0 & \begin{matrix} 0 & 0 \\ 1 & -\gamma \end{matrix} \end{bmatrix},$$

$$C = \begin{bmatrix} 0_{(k+1)\times 1} \\ \tau \\ 0 \end{bmatrix}, \; D = \begin{bmatrix} 1 & 0_{1\times(k+2)} & \\ I_{k\times k} & 0_{k\times 3} & \\ 0_{1\times(k+1)} & 1 & 0 \\ 0_{1\times(k+1)} & 1 & 0 \end{bmatrix}, \; E = \begin{bmatrix} 1 & 0 \\ 0_{k\times 1} & 0_{k\times 1} \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$\zeta_t = \begin{bmatrix} \varepsilon_t^{\pi} \\ \varepsilon_t^{o} \end{bmatrix}, \; \eta_t = \begin{bmatrix} \varepsilon_t^{u} \\ \varepsilon_t^{y} \end{bmatrix}.$$

5.2 Estimation Method

The Bayesian methodology involves formulating prior distributions for variables of interest and deriving estimates from the posterior distributions formed from the likelihood functions and the priors. This essentially gives a balance between prior beliefs and the data information embodied in the sample.

MCMC method is an iterative simulation algorithm that facilitates Bayesian analysis. Sequential drawing/sampling of estimates from the conditional probability functions of the parameters are performed iteratively, and the ergodic averages of these draws are used for statistical inference. More details on implementation of the algorithm can be found, for example, in Gelman et al. (2000).

In our context, prior distributions for the unknown parameters A, B, C and Ω and Ψ have to be pre-determined. The priors are mostly conjugate, with normal distributions chosen for the non-zero elements of $A, B,$ and C. Ω and Ψ are distributed according to the inverted-Wishart distributions. Given the likelihood functions, A, B, Ω and Ψ can be sampled using the Gibbs sampler. Updates of C are performed using a Metropolis Hastings step, so is the sampling of the state vector θ (see details in Geweke and Tanizaki (2001)). We set for a burn-in of 50,000 iterations and a sampling size of the same amount.